TEKS Cluster: Coordinate and Transformational Geometry

G.2 **Coordinate and transformational geometry.** The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures.

G.3 **Coordinate and transformational geometry.** The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity).

Connected Knowledge and Skills G.12

Coordinate Geometry

Readiness Standards

G.2(B) derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines

G.2(C) determine an equation of a line parallel or perpendicular to a given line that passes through a given point

Supporting Standards

G.2(A) determine the coordinates of a point that is a given fractional distance less than one from one end of a line segment to the other in one- and two-dimensional coordinate systems, including finding the midpoint

G.12(E) show that the equation of a circle with center at the origin and radius r is $x^2 + y^2 = r^2$ and determine the equation for the graph of a circle with radius r and center (h, k), $(x - h)^2 + (y - k)^2 = r^2$

Transformations

Readiness Standards

G.3(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane

Supporting Standards

G.3(A) describe and perform transformations of figures in a plane using coordinate notation

G.3(C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane

G.3(D) identify and distinguish between reflectional and rotational symmetry in a plane figure

TEKS Scaffold

TEKS	Student Expectation
G.2(B)	**G.2 Coordinate and transformational geometry.** The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to: **(B) derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines**
8.7(D)	determine the distance between two points on a coordinate plane using the Pythagorean theorem (S)
8.4(A)	use similar right triangles to develop an understanding that slope, *m*, given as the rate comparing the change in *y*-values to the change in *x*-values, $(y_2 - y_1)/(x_2 - x_1)$, is the same for any two points (x_1, y_1) and (x_2, y_2) on the same line (S)
8.6(C)	use models and diagrams to explain the Pythagorean theorem (S)

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

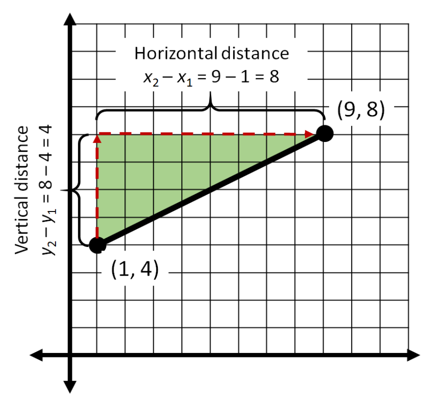
Content Builder (see Appendix for Tree Diagram)

- Derive the distance formula
- Use the distance formula to verify geometric relationships including the congruence of segments
- Derive the slope formula
- Use the slope formula to verify geometric relationships including parallelism and perpendicularity of lines
- Derive the midpoint formula
- Use the midpoint formula to verify geometric relationships

Instructional Implications

Instruction should include activities where students derive and then use the distance formula ($d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$), slope formula ($m = \frac{y_2 - y_1}{x_2 - x_1}$), and midpoint formula ($M = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$) to verify geometric relationships, (including the congruence of segments and the parallelism or perpendicularity of pairs of lines).

To derive formulas, students must first relate the horizontal change between the two points as the difference between the *x*-coordinates, and the vertical change between two points as the difference between the *y*-coordinates. From this, the distance formula may then be established by applying the Pythagorean Theorem to the horizontal and vertical lengths to determine the length of the hypotenuse (which is also the distance between two points) of the right triangle formed. Similarly, for the slope formula, instruction should include the calculation for slope as the ratio of the vertical change to the horizontal change.



Based on the Pythagorean Theorem ($a^2 + b^2 = c^2$, or $c = \sqrt{a^2 + b^2}$), the distance between two points is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Here, } d = \sqrt{(9 - 1)^2 + (8 - 4)^2}$$

$$d = \sqrt{(8)^2 + (4)^2} = \sqrt{80} = 4\sqrt{5}$$

$$\text{Slope} = m = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

$$\text{Here, slope} = \frac{8 - 4}{9 - 1} = \frac{4}{8} = \frac{1}{2}$$

(continued)

Instructional Implications (continued)

Instruction should also include the derivation of the midpoint formula. To find the location of a point *M* on a number line that is halfway between two other points (x_1 and x_2), one method is to:

- Subtract the coordinates for x_1 and x_2 to find the distance between them
- Divide that distance by 2
- Add the result to the first coordinate

Or, use $M = x_1 + (x_2 – x_1)/2$. This expression, when simplified, is equivalent to the first part of the midpoint formula, $\frac{x_1 + x_2}{2}$. The process can be repeated to include the *y*-coordinates on the coordinate grid.

Finally, instruction should include the use of the slope, distance, and midpoint formulas to verify geometric relationships (e.g., given quadrilateral *ABCD* with vertices, *A*(3, 4), *B*(7, 8), *C*(9, 6) and *D*(5, 2), prove it is a rectangle and determine the coordinates of the midpoint of the diagonals of the quadrilateral).

Learning from Mistakes

Students may make the following mistakes:

- Substituting the *x*- and *y*-values incorrectly when using the formulas (substituting *y*-values in for *x*-values)
- When using the distance formula, dividing a value by "2" instead of taking the square root
- Adding the *x*-value to the *y*-value, instead of computing the sum of the *x*-values and computing the sum of the *y*-values before dividing by 2 in the midpoint formula
- Incorrectly writing the ratio of the slope of a line as the ratio of horizontal change divided by vertical change ($\frac{x_2 - x_1}{y_2 - y_1}$)

Academic Vocabulary

congruent
distance formula
line
midpoint formula
parallel
perpendicular
segment
slope formula

TEKS Scaffold

TEKS	Student Expectation
G.2(C)	**G.2 Coordinate and transformational geometry.** The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to: **(C) determine an equation of a line parallel or perpendicular to a given line that passes through a given point**
A.2(E)	write the equation of a line that contains a given point and is parallel to a given line (S)
A.2(F)	write the equation of a line that contains a given point and is perpendicular to a given line (S)
8.5(I)	write an equation in the form $y = mx + b$ to model a linear relationship between two quantities using verbal, numerical, tabular, and graphical representations (R)

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

linear equation
parallel lines
perpendicular lines
point-slope form
slope
slope-intercept form
standard form
y-intercept

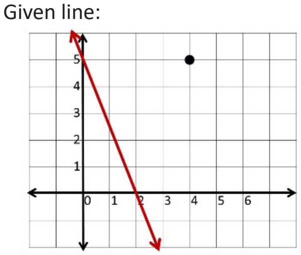
Content Builder (see Appendix for Tree Diagram)

- Determine an equation of a line parallel to a given line that passes through a given point
- Determine an equation of a line perpendicular to a given line that passes through a given point

Instructional Implications

Students are expected to determine the equation of a line parallel (equal slopes) or perpendicular (slopes that are opposite reciprocals) to a given line that passes through a given point. Instruction should include equations of lines written in slope-intercept form ($y = mx + b$), standard form ($Ax + By = C$, where the slope is equal to $-\frac{A}{B}$), and lines graphed on a coordinate plane. Refer to the examples below.

For each example use the point-slope form [$y - y_1 = m(x - x_1)$] to determine the equations of the lines through the point (4, 5) that are parallel and perpendicular to the given line.

Example A	Example B	Example C
Given line: $y = \frac{3}{4}x + 2$ Point: (4, 5)	Given line: $2x - 3y = -7$ Point: (4, 5)	Given line:  Point: (4, 5)

Solutions

	Example A	Example B	Example C
Parallel line:	slope = $\frac{3}{4}$ $y - 5 = \frac{3}{4}(x - 4)$ $y = \frac{3}{4}x + 2$	slope = $-\left(\frac{2}{-3}\right) = \frac{2}{3}$ $y - 5 = \frac{2}{3}(x - 4)$ $y = \frac{2}{3}x + \frac{7}{3}$	slope = $-\frac{5}{2}$ $y - 5 = -\frac{5}{2}(x - 4)$ $y = -\frac{5}{2}x + 15$
Perpendicular line:	slope = $-\frac{4}{3}$ $y - 5 = -\frac{4}{3}(x - 4)$ $y = -\frac{4}{3}x + \frac{31}{3}$	slope = $-\frac{3}{2}$ $y - 5 = -\frac{3}{2}(x - 4)$ $y = -\frac{3}{2}x + 11$	slope = $\frac{2}{5}$ $y - 5 = \frac{2}{5}(x - 4)$ $y = \frac{2}{5}x + \frac{17}{5}$

Learning from Mistakes

Students may make the following mistakes:

- Using the slope formula incorrectly ($\frac{\text{horizontal change}}{\text{vertical change}}$ instead of $\frac{\text{vertical change}}{\text{horizontal change}}$)
- Thinking the slopes of perpendicular lines are only opposite values instead of opposite reciprocals

G.2(A) **G.2 Coordinate and transformational geometry.** The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to:

(A) determine the coordinates of a point that is a given fractional distance less than one from one end of a line segment to the other in one- and two-dimensional coordinate systems, including finding the midpoint

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

coordinate
distance
endpoint
line segment
midpoint
point

Role in Concept Development

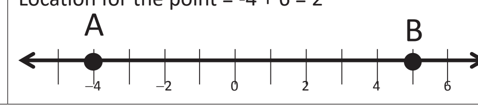
Supports
G.2(B) derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines

Connection/ Relevance
This skill connects to the derivation and application of the midpoint formula. Specifically, the midpoint formula can be derived as a special case where the "fractional distance" is $\frac{1}{2}$; or, the midpoint formula can be extended to include other fractions besides $\frac{1}{2}$.

When to Teach
With G.2(B)

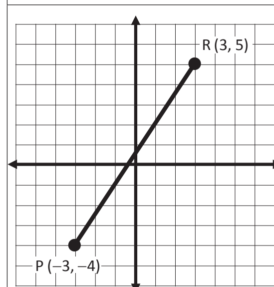
Role in Concept Development (continued)

Instructional Implications
Students are expected to determine the coordinates of a point that is a given fractional distance from either endpoint of a line segment. For one-dimensional coordinate systems (number lines), this process can be demonstrated by: subtracting the coordinates of the endpoints to determine the distance between; multiplying the distance by the fraction; and adding or subtracting this partial distance to the location or coordinates of the initial endpoint. (See the example below.)

Find the location of the point $\frac{2}{3}$ of the distance from point *A* to point *B*.	*A* and *B* have respective coordinates of -4 and 5. Distance from *A* to *B* = 5 - (-4) = 9 $\frac{2}{3}$ of the distance from *A* to *B* = $\frac{2}{3}$(9) = 6 Location for the point = -4 + 6 = 2 

From such examples, instruction should include the following formula:

- The location on a number line that is a fractional distance, k (where $0 < k < 1$) from one endpoint, x_1 , to another, x_2, can be determined using the expression $x_1 + k\,(x_2 - x_1)$.
- This formula can be extended to two-dimensional coordinate systems: The location on a line segment that is a fractional distance, k (where $0 < k < 1$) from one endpoint (x_1, y_1), to another (x_2, y_2), can be determined using the expression $(x_1 + k(x_2 - x_1), y_1 + k(y_2 - y_1))$. See the example below.

Find the location of the point $\frac{1}{3}$ of the distance from point *R* to point *P*.	
 	Here, (x_1, y_1) = (3, 5); (x_2, y_2) = (-3, -4); and $k = \frac{1}{3}$. Using $(x_1 + k(x_2 - x_1), y_1 + k(y_2 - y_1))$, $(3 + \frac{1}{3}(-3 - 3), 5 + \frac{1}{3}(-4 - 5))$ $(3 + \frac{1}{3}(-6), 5 + \frac{1}{3}(-9))$ (3 + (-2), 5 + (-3)) (1, 2)

Learning from Mistakes
Students may make the following mistakes:

- Incorrectly identifying the point at which to begin [to use as (x_1, y_1)]
- Making arithmetic mistakes with fractions or subtractions

G.12 Circles. The student uses the process skills to understand geometric relationships and apply theorems and equations about circles. The student is expected to:

G.12(E) **(E)** show that the equation of a circle with center at the origin and radius r is $x^2 + y^2 = r^2$ and determine the equation for the graph of a circle with radius r and center (h, k), $(x - h)^2 + (y - k)^2 = r^2$

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

center of a circle
circle
radius

Role in Concept Development

Supports

G.2(B) derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of line

Connection/ Relevance

The equation for a circle is a direct extension of the distance formula.

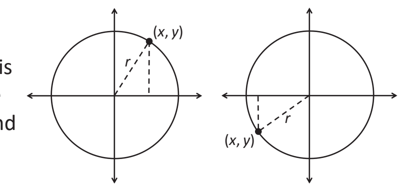
Role in Concept Development (continued)

When to Teach

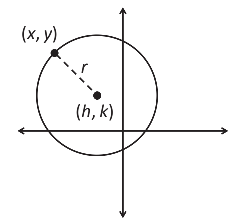
After G.2(B)

Instructional Implications

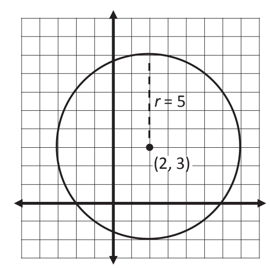
Instruction should include the development of the equation for a circle on the coordinate grid.



A circle can be defined as the set of all points in a plane that are equal distance (r) from a center point. When the circle is centered at the origin on the coordinate plane, the points on the circle have x- and y-values that form a right triangle with r as the hypotenuse. The Pythagorean Theorem may be applied to each right triangle formed to derive the equation of a circle as $x^2 + y^2 = r^2$.



When a circle on the coordinate grid is not centered at the origin, the distance formula ($d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$) may be used to determine the equation for the graph of a circle. For example, consider the circle in the diagram below with radius r and center (h, k).



Since each point on circle is the same distance (r) from the center (h, k), applying the distance formula yields the equation $r = \sqrt{(x - h)^2 + (y - k)^2}$). This can be re-written as the standard equation for the graph of a circle: $(x - h)^2 + (y - k)^2 = r^2$.

Students should then be able to write the equation for a circle from the graph. In the example provided, the circle is centered at (2, 3) with a radius of 5 units. So, the equation would be: $(x - 2)^2 + (y - 3)^2 = 25$.

Learning from Mistakes

Students may make the following mistakes:

- Using addition instead of subtraction for the center's coordinates in the equation for a circle
- Forgetting to include the exponents of 2 in the equation for a circle
- Using diameter instead of radius, or for the value of r^2

TEKS Scaffold

TEKS	Student Expectation
G.3(B)	**G.3 Coordinate and transformational geometry.** The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and nonrigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to: **(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane**
G.3(A)	describe and perform transformations of figures in a plane using coordinate notation (S)
8.10(C)	explain the effect of translations, reflections over the x- or y-axis, and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation (R)
8.10(B)	differentiate between transformations that preserve congruence and those that do not (S)

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

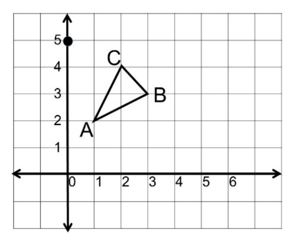
center of dilation
composition
dilation
enlargement (scale factor > 1)
image
non-rigid transformation
point (or center) of rotation
pre-image
reduction (0 < scale factor < 1)
reflection
rigid transformation
rotation
scale factor
similarity
transformation
translation

Content Builder (see Appendix for Tree Diagram)

- Determine the image of a given two-dimensional figure under a composition of rigid transformations
- Determine the pre-image of a given two-dimensional figure under a composition of rigid transformations
- Determine the image of a given two-dimensional figure under a composition of non-rigid transformations
- Determine the pre-image of a given two-dimensional figure under a composition of rigid transformations
- Determine the image of a given two-dimensional figure under a composition of rigid and non-rigid transformations
- Determine the pre-image of a given two-dimensional figure under a composition of rigid and non-rigid transformations
- Determine the image of a given two-dimensional figure that includes dilations where the center can be any point in the plane
- Determine the pre-image of a given two-dimensional figure that includes dilations where the center can be any point in the plane

Instructional Implications

Students are expected to use both rigid transformations (translations, rotations, reflections) and non-rigid transformations (dilations)—as well as compositions of all these types—to determine the image (a figure after a transformation) or pre-image (the original figure before a transformation). These transformations can be represented symbolically, with rules such as $(x, y) \rightarrow (-(x - 1), y + 2)$, or verbally, with descriptions such as, "Translate the figure one unit to the left and two units up, then reflect it over the *y*-axis." Instruction should include examples such as in the charts below.

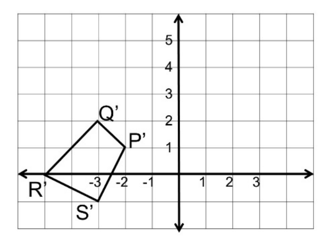


Triangle *ABC* in the figure above is to be dilated through the center point (0, 5) with a scale factor of 2.

Or, $(x, y) \rightarrow (2x, 2(y - 5) + 5)$

In the image figure, what will be the coordinates of the vertices *A*', *B*', and *C*'?

Answer: A'(2, -1), B'(6,1), C'(4, 3)



A figure *PQRS* is translated two units to the right and three units down, then reflected over the y-axis. The result (image) of this composite transformation is shown in the figure.

What are the coordinates for vertices (*P*, *Q*, *R*, *S*) of the original figure (pre-image)?

Answer: P(0, 4), Q(1, 5), R(3, 3), S(1, 2)

Learning from Mistakes

Students may make the following mistakes:

- Not being able to distinguish the difference between image and pre-image
- Thinking the origin is the only point that can be the center for dilations

G.3(A) **G.3 Coordinate and transformational geometry.** The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:

(A) describe and perform transformations of figures in a plane using coordinate notation

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

center of dilation
coordinate notation
dilation
enlargement (scale factor > 1)
image
non-rigid transformation
point (or center) of rotation
pre-image
reduction (0 < scale factor < 1)
reflection
rigid transformation
rotation
scale factor
similarity
transformation
translation

Role in Concept Development

Supports

G.3(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane

Connection/ Relevance

Coordinate notation provides the symbolic way to describe transformations.

When to Teach

Before G.3(B)

Instructional Implications

Instruction should include using coordinate notation to describe a variety of examples of transformations of figures in a plane, including reflections, rotations, translations, and dilations. The table below shows some examples describing different transformations using coordinate notation.

Verbal description	Coordinate notation
Reflection across the *x*-axis	$(x, y) \rightarrow (x, -y)$
Reflection across the *y*-axis	$(x, y) \rightarrow (-x, y)$
Rotation 90° counter-clockwise about the origin	$(x, y) \rightarrow (-y, x)$
Translation 4 units left and 3 units up	$(x, y) \rightarrow (x - 4, y + 3)$
Dilation with a scale factor of *k*, centered at the origin	$(x, y) \rightarrow (kx, ky)$
Dilation with a scale factor of *k*, centered at the point (*a*, *b*)	$(x, y) \rightarrow (k(x - a) + a, k(y - b) + b)$

- Given a pre-image figure and coordinate notation, students should be able to generate the image (or the coordinates for the vertices of the image).
- Given a verbal description of a transformation, students should be able to generate the coordinate notation.
- Given the coordinate notation, students should be able to provide a verbal description of the transformation.

Learning from Mistakes

Students may make the following mistakes:
- Confusing or interchanging the *x* and *y* coordinates when using formulas
- Incorrectly identifying the type of transformation (reflect instead of rotate, left instead of down, etc.)

G.3(C) Supporting

G.3(C)

G.3 Coordinate and transformational geometry. The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:

(C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

dilation
enlargement (scale factor > 1)
image
point (or center) of rotation
pre-image
reduction (0 < scale factor < 1)
reflection
rotation
scale factor
similarity
transformation
translation

Role in Concept Development

Supports

G.3(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane

Connection/ Relevance

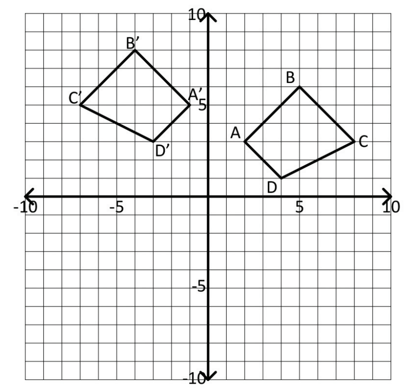
Identifying the sequence of transformations is relevant in determining the image or pre-image of various transformations.

When to Teach

With G.3(B)

Instructional Implications

Instruction should include a variety of examples where students identify the sequence of transformations that carry a pre-image onto an image on and off the coordinate plane.



For example, in the figure at right, students might recognize several different methods for transforming the quadrilateral *ABCD* (pre-image) to the image *A′B′C′D′*.

- Translate 2 units up, then reflect over the y-axis, then translate 1 unit to the right.
- Translate one unit to the left, then translate 2 units up, then reflect over the y-axis.
- Reflect over the y-axis, then translate 2 units up and 1 unit to the right.

Learning from Mistakes

Students may make the following mistakes:
- Confusing or interchanging which figure is the image and which is the pre-image
- Placing the sequence of transformations in the wrong order (where order matters)

G.3(D) Supporting

G.3(D) **G.3 Coordinate and transformational geometry.** The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:

(D) identify and distinguish between reflectional and rotational symmetry in a plane figure

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

degree (or angle) of rotation
horizontal line of reflection
line of symmetry
plane figure
point (or center) of rotation
reflectional symmetry
rotational symmetry
vertical line of reflection

Role in Concept Development

Supports

G.3(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane

Connection/ Relevance

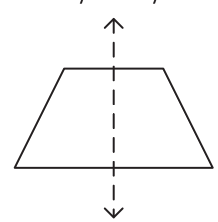
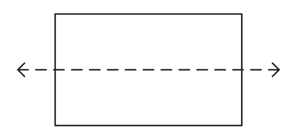
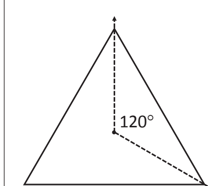
Identifying symmetry in a single plane figure helps solidify concepts like reflection (horizontal or vertical) and rotation (by a certain number of degrees). These concepts are integral in describing transformations that generate new (image) figures.

When to Teach

Before G.3(B)

Instructional Implications

Instruction should include a variety of examples where students identify and distinguish between reflectional symmetry (where half of a figure is a mirror image of the other half of the figure over a line of reflection) and rotational symmetry (where rotating a figure about a point of rotation results in an image that is identical to the pre-image) in a plane figure. Some examples are shown below.

Vertical Reflectional Symmetry	Horizontal Reflectional Symmetry	Rotational Symmetry
		

Learning from Mistakes

Students may make the following mistakes:

- Not identifying all of a figure's lines of symmetry (when multiple symmetries are present)
- When describing rotational symmetry, forgetting to include the degree measure

