

TEKS Cluster: Geometry and Measurement – Three-Dimensional

- 8.6 Expressions, equations, and relationships.** The student applies mathematical process standards to develop mathematical relationships and make connections to geometric formulas.
- 8.7 Expressions, equations, and relationships.** The student applies mathematical process standards to use geometry to solve problems.

Volume

Readiness Standards

8.7(A) solve problems involving the volume of cylinders, cones, and spheres

Supporting Standards

8.6(A) describe the volume formula $V = Bh$ of a cylinder in terms of its base area and its height

Non-tested Standards

8.6(B) model the relationship between the volume of a cylinder and a cone having both congruent bases and heights and connect that relationship to the formulas

Surface Area

Readiness Standards

8.7(B) use previous knowledge of surface area to make connections to the formulas for lateral and total surface area and determine solutions for problems involving rectangular prisms, triangular prisms, and cylinders

TEKS Scaffold

TEKS	Student Expectation
G.11(D)	apply the formulas for the volume of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure (R)

8.7 Expressions, equations, and relationships. The student applies mathematical process standards to use geometry to solve problems.

8.7(A) The student is expected to:

(A) solve problems involving the volume of cylinders, cones, and spheres

8.6(A)	describe the volume formula $V = Bh$ of a cylinder in terms of its base area and its height (S)
7.9(B)	determine the circumference and area of circles (R)
7.9(A)	solve problems involving the volume of rectangular prisms, triangular prisms, rectangular pyramids, and triangular pyramids (R)

Stimulus

Word Problem*	Verbal Description*	Chart/Table	Graph
Equation/Expression	Ordered Pairs	Diagram/Image*	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures*

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Content Builder (see Appendix for Tree Diagram)

- Solve volume problems involving cylinders
- Solve volume problems involving cones
- Solve volume problems involving spheres

Instructional Implications

In conjunction with 8.6(A)/(B), students determine the solution for problems involving the volume of cylinders, cones, and spheres. Problems should include positive rational numbers (decimals and fractions).

Instruction should vary the context of the problems, for example:

- Given the lengths of radius/diameter/height, determine the volume
- Given the volume and one of the dimensions of the radius/diameter and/or height, determine the missing radius/diameter and/or height
- Given the area of the base, determine the missing height

It is important that students understand why length is represented in units, area is represented in square units, and volume is represented in cubic units.

Instruction should include the use of the π key on a calculator, 3.14, or $\frac{22}{7}$ when solving problems involving the volume of cylinders, cones, and spheres. Students should be expected to represent solutions in terms of π or as a rational number approximation.

Learning from Mistakes

Students may make the following mistakes:

- Confusing the diameter with the radius*
- Confusing the slant height of a cone with the height of a cone
- Not understanding the “ B ” in the formula $V = Bh$ or $V = \frac{1}{3} Bh$ represents the area of the base of the shape*
- Using the formula for the circumference of a circle instead of the formula for the area of a circle when calculating the area of the base of a cylinder or cone*
- Forgetting to take $\frac{1}{3}$ the value of the Bh when determining the volume of a cone/pyramid*

Academic Vocabulary

formula (cone/pyramid volume): $V = \frac{1}{3} Bh$

formula (cylinder/prism volume): $V = Bh$ *

formula (sphere volume): $V = \frac{4}{3} \pi r^3$ *

π (pi)
area of the base
radius*

Interesting Items

8.7(A) 2023 #14

8.7(A) 2022 #33

8.7(A) 2021 #18

8.7(A) 2019 #31

8.7(A) 2018 #25

8.6 Expressions, equations, and relationships. The student applies mathematical process standards to develop mathematical relationships and make connections to geometric formulas. The student is expected to:

(A) describe the volume formula $V = Bh$ of a cylinder in terms of its base area and its height

Role in Concept Development

- Supports** 8.7(A) solve problems involving the volume of cylinders, cones, and spheres
- Connection/Relevance** Being able to describe the volume formula of a cylinder in terms of its base area and its height provides the foundation students need to solve problems involving the volume of a cylinder.
- When to Teach** Before/Prerequisite to 8.7(A)
- Instructional Implications** In conjunction with 7.8(C), students build on their prior experience of the area formula of a circle to describe the volume formula of a cylinder ($V = Bh$) in terms of its base area, B (i.e. area of a circle where $A = \pi r^2$) and its height, h .
Instruction should include experiences where students verbally describe this relationship (e.g., the formula for the volume of a cylinder is the area of a circle, since the base of a cylinder is in the shape of a circle multiplied by the height of the cylinder since a cylinder is like layers of congruent circles). This process should be repeated for several different cylinders.
Students should be expected to describe the volume of a cylinder in terms of π or as a rational number approximation.
- Learning from Mistakes** Students may make the following mistakes:
 - Confusing the diameter with the radius*
 - Multiplying the radius of the circular base of a cylinder by 2 instead of squaring it*
 - Not understanding how to calculate the height from the given volume*

Stimulus

Word Problem*	Verbal Description	Chart/Table	Graph
Equation/Expression*	Ordered Pairs	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula*	Geometric Figures*

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

- π (pi)
- area of the base*
- diameter*
- formula (cylinder/prism volume): $V = Bh$ *
- height*
- radius*
- volume (cylinder)*

Interesting Items

- 8.6(A) 2023 #31
- 8.6(A) 2021 #14
- 8.6(A) 2016 #29

8.6 Expressions, equations, and relationships. The student applies mathematical process standards to develop mathematical relationships and make connections to geometric formulas.

8.6(B) The student is expected to:

(B) model the relationship between the volume of a cylinder and a cone having both congruent bases and heights and connect that relationship to the formulas

Role in Concept Development

- Supports** 8.7(A) solve problems involving the volume of cylinders, cones, and spheres
- Connection/Relevance** Being able to model the relationship between the volume of a cylinder and a cone having congruent bases and heights provides the foundation students need to solve problems involving the volume of cylinders and cones.
- When to Teach** Before/Prerequisite to 8.7(A)
- Instructional Implications** Students model the relationship of the volume of a cylinder and a cone with congruent bases and heights and connect this relationship to the formulas for each.
 - $V = Bh$ and $V = \frac{1}{3} Bh$ are the formulas for a cylinder and cone, respectively
 - In both formulas, B represents the area of the base, which is the formula for the area of a circle (πr^2)
 - Therefore, the equivalent formula for a cylinder is $V = \pi r^2 h$, and the equivalent formula for a cone is $V = \frac{1}{3} \pi r^2 h$

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/Expression	Ordered Pairs	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

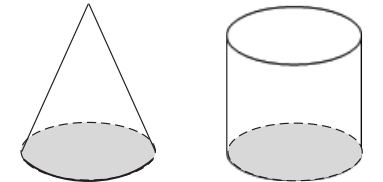
Academic Vocabulary

formula (cone/pyramid volume): $V = \frac{1}{3} Bh$

formula (cylinder/prism volume): $V = Bh$

π (pi)
area of the base
height
radius

Instruction should include experiences where students verify congruent bases and heights, then compare the volume of each. Students should fill a cone (e.g., with water, sand, rice, etc.), and then empty the contents into a cylinder to determine the relationship and vice versa (e.g., the volume of a cylinder is three times the volume of a cone with a base and height congruent to the cylinder, or the volume of the cone is one-third the volume of a cylinder). This process should be repeated for several different cylinders and cones.



The volume of 3 cones with a base and height congruent = to the cylinder.

The volume of 1 cylinder with a base and height congruent to the cone.

Learning from Mistakes

- Students may make the following mistakes:
- Confusing the height of the cone with the lateral side length of a triangular face

TEKS Scaffold

TEKS	Student Expectation
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8.7 Expressions, equations, and relationships. The student applies mathematical process standards to use geometry to solve problems. The student is expected to:

- 8.7(B) **(B) use previous knowledge of surface area to make connections to the formulas for lateral and total surface area and determine solutions for problems involving rectangular prisms, triangular prisms, and cylinders**

7.9(D)	solve problems involving the lateral and total surface area of a rectangular prism, rectangular pyramid, triangular prism, and triangular pyramid by determining the area of the shape's net (S)
7.9(C)	determine the area of composite figures containing combinations of rectangles, squares, parallelograms, trapezoids, triangles, semi-circles, and quarter circles (R)
7.9(B)	determine the circumference and area of circles (R)

Stimulus

Word Problem*	Verbal Description	Chart/Table*	Graph
Equation/Expression	Ordered Pairs	Diagram/Image*	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures*

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor* (1-2 pts)	Multiple Choice* (1 pt)	

Content Builder (see Appendix for Tree Diagram)

- Use knowledge of surface area to make connections to formulas for lateral surface area
- Use knowledge of surface area to make connections to formulas for total surface area
- Determine solutions for problems involving rectangular prisms
- Determine solutions for problems involving triangular prisms
- Determine solutions for problems involving cylinders

Instructional Implications

In conjunction with 7.9(D), students make connections to the formulas for lateral and total surface area and determine solutions for problems involving rectangular prisms, triangular prisms, and cylinders. Instruction should include a variety of problem situations that include positive rational numbers (decimals and fractions). Instruction should vary the context of the problems, for example:

- Given the dimensions, determine the lateral area and/or total surface area
- Given the lateral area and one of the dimensions of the sides/edges/radius/diameter and/or height, determine the missing side/edge/radius/diameter and/or height

Instruction should include examples of when to apply lateral versus total surface area (e.g., a canned good label would represent the lateral area, and the metal part of the can would represent the total surface area). Instruction should include the use of the π key on a calculator, 3.14, or $\frac{22}{7}$ when solving problems involving the lateral or total surface area of rectangular prisms, triangular prisms, or cylinders.

Learning from Mistakes

Students may make the following mistakes:

- Not understanding the "B" in the formula $S = Ph + 2B$ represents the area of the base of the shape*
- Not understanding the "P" in the formula $S = Ph$ and/or $S = Ph + 2B$ represents the perimeter/circumference of the base of the shape*
- Using the formula for the circumference of a circle instead of the formula for the area of a circle when calculating the area of the base of a cylinder
- Having difficulty envisioning three-dimensional solids when no image or only one face of the solid is given*
- Having difficulty working backwards to determine the length of a missing side when given the lateral/total surface area*

Academic Vocabulary

- formulas (lateral surface area):
- $S = Ph$
 - $S = 2\pi rh$
- formulas (total surface area):
- $S = Ph + 2B$
 - $S = 2\pi rh + 2\pi r^2$

- circumference*
- lateral surface area*
- perimeter of the base
- total surface area*

Interesting Items

- 8.7(B) 2024 #8
- 8.7(B) 2023 #6
- 8.7(B) 2023 #25
- 8.7(B) 2018 #9
- 8.7(B) 2017 #38