# TEKS Cluster: Proportional and Non-Proportional Reasoning

**8.4** **Proportionality.** The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope.

**8.5** **Proportionality.** The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions.

**8.9** **Expressions, equations, and relationships.** The student applies mathematical process standards to use multiple representations to develop foundational concepts of simultaneous linear equations.

## Slope

***Readiness Standards***

8.4(B) graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship

***Supporting Standards***

8.4(A) use similar right triangles to develop an understanding that slope, *m*, given as the rate comparing the change in *y*-values to the change in *x*-values, $(y_2 - y_1)/(x_2 - x_1)$, is the same for any two points $(x_1, y_1)$ and $(x_2, y_2)$ on the same line

## Proportional Reasoning

***Supporting Standards***

8.5(A) represent linear proportional situations with tables, graphs, and equations in the form of $y = kx$

8.5(E) solve problems involving direct variation

## Non-Proportional Reasoning

***Readiness Standards***

8.4(C) use data from a table or graph to determine the rate of change or slope and *y*-intercept in mathematical and real-world problems

8.5(I) write an equation in the form $y = mx + b$ to model a linear relationship between two quantities using verbal, numerical, tabular, and graphical representations

***Supporting Standards***

8.5(B) represent linear non-proportional situations with tables, graphs, and equations in the form of $y = mx + b$, where $b \neq 0$

8.9(A) identify and verify the values of *x* and *y* that simultaneously satisfy two linear equations in the form $y = mx + b$ from the intersections of the graphed equations

## Proportional and Non-Proportional Recognition

***Supporting Standards***

8.5(F) distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form $y = kx$ or $y = mx + b$, where $b \neq 0$

8.5(H) identify examples of proportional and non-proportional functions that arise from mathematical and real-world problems

## Function Identification

***Readiness Standards***

8.5(G) identify functions using sets of ordered pairs, tables, mappings, and graphs

# 8.4(B) Readiness

Subcluster: Slope

## TEKS Scaffold

| TEKS | Student Expectation |
|---|---|
| A.3(B) | calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in contents of mathematical and real-world problems (S) |
| 8.4(C) | use data from a table or graph to determine the rate of change or slope and y-intercept in mathematical and real-world problems (R) |
| 8.4(B) | **8.4 Proportionality.** The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope. The student is expected to: **(B) graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship** |
| 7.4(A) | represent constant rates of change in mathematical and real-world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including $d = rt$ (R) |
| 7.4(C) | determine the constant of proportionality ($k = y/x$) within mathematical and real-world problems (S) |
| 6.6(C) | represent a given situation using verbal descriptions, tables, graphs, and equations in the form $y = kx$ or $y = x + b$ (R) |

## Stimulus

| | | | |
|---|---|---|---|
| Word Problem* | Verbal Description* | Chart/Table | Graph* |
| Equation/ Expression | Ordered Pairs | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

## Item Types

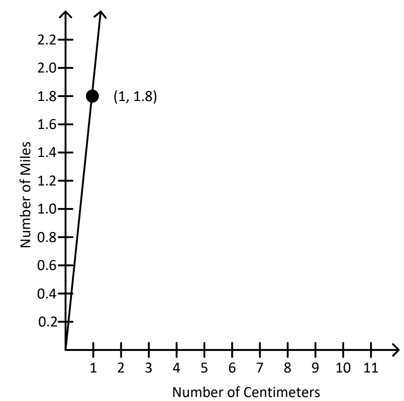
| | | | |
|---|---|---|---|
| Multiselect (2 pts) | Match Table Grid (2 pts) | Drag and Drop (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

## Content Builder (see Appendix for Tree Diagram)

- Graph proportional relationships and interpret the unit rate as the slope of the line that models the relationship

## Instructional Implications

Instruction should include a variety of problems where students graph proportional relationships and interpret the unit rate ($k = y/x$ simplified to $x = 1$) as the slope of the line that models the relationship.




For example:

- the scale on a map shows 5 centimeters and represents the actual distance of 9 miles
- the rate $\frac{9\text{ miles}}{5\text{ centimeters}}$ is equivalent to the unit rate $\frac{1.8\text{ miles}}{1\text{ centimeter}}$
- the unit rate is the slope (1.8) for the line ($y = 1.8x$)
- the slope is interpreted as an increase of 1 cm on the map, which is an increase of 1.8 miles in actual distance (as shown in the graph at right)

Instruction should make the connection between unit rate, slope, and the constant of proportionality.

## Learning from Mistakes

Students may make the following mistakes:

- Graphing a non-proportional relationship and interpreting the slope as the unit rate
- Not understanding a linear proportional relationship goes through the origin
- Struggling when calculating the unit rate due to looking only at the change in the *y* values and ignoring the change in the *x* values
- Interpreting the unit rate and slope as $k = x/y$ instead of $k = y/x$*

## Academic Vocabulary

slope*, *m*
unit rate*

## Interesting Items

8.4(B) 2024 #11
8.4(B) 2016 #5

**8.4 Proportionality.** The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope. The student is expected to:

8.4(A) **(A) use similar right triangles to develop an understanding that slope, *m*, given as the rate comparing the change in *y*-values to the change in *x*-values, $(y_2 - y_1)/(x_2 - x_1)$, is the same for any two points $(x_1, y_1)$ and $(x_2, y_2)$ on the same line**

## Stimulus

| | | | |
|---|---|---|---|
| Word Problem | Verbal Description* | Chart/Table | Graph* |
| Equation/ Expression* | Ordered Pairs | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures* |

## Item Types

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| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

## Academic Vocabulary

hypotenuse*
proportion*
ratio*
similar right triangle*
slope*, *m*

## Interesting Items

8.4(A) 2023 #5
8.4(A) 2021 #15
8.4(A) 2019 #18
8.4(A) 2017 #30

## Role in Concept Development

**Supports**

8.4(B) graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship

**Connection/ Relevance**

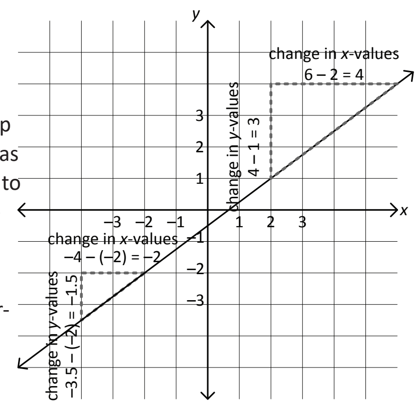
Using similar right triangles to develop an understanding for the slope of a line is the foundation for students to graph proportional relationships and interpret the unit rate as the slope of the line.

**When to Teach**

Before/Prerequisite to 8.4(B)

**Instructional Implications**

Instruction should include problems where students use similar right triangles and their coordinates to develop an understanding that slope, *m*, given as the rate comparing change in *y*-values to change in *x*-values (e.g., $\frac{y_2 - y_1}{x_2 - x_1}$ ) is the same for any two points on that line.




Consider the graph at right where the two right triangles are similar since corresponding sides are proportional and corresponding angles are congruent. The two triangles model the slope, *m*, of the line as the rate comparing the change of the *y*-values to the change of the *x*-values (e.g., $\frac{-3.5-(-2)}{-4-(-2)} = \frac{-1.5}{-2} = \frac{3}{4}$ which is equivalent to $\frac{4-1}{6-2} = \frac{3}{4}$ ).

Teaching the formula for slope is not the intent of this standard. Rather, students should understand that comparing the change in *y*- and *x*-values is the same for any two points on the same line. This includes the concept of reversing the order of the points.

**Learning from Mistakes**

Students may make the following mistakes:

- Having difficulty establishing the correct proportion to relate to the slope of the line*
- Not realizing that the longest side (hypotenuse) of the similar right triangles must be situated on the same line to have the same slope
- Assuming that the similar right triangles have to be on the same side of the line in order for the slope of the line to be the same
- Determining the slope of a line as the change in the *x*-values over the change in the *y*-values
- Incorrectly determining the difference in *x*- or *y*-values when the ordered pair includes a negative value*

 * Used on STAAR 

8.5(A) **8.5 Proportionality.** The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to:

**(A) represent linear proportional situations with tables, graphs, and equations in the form of $y = kx$**

## Stimulus

| | | | |
|---|---|---|---|
| Word Problem* | Verbal Description | Chart/Table* | Graph |
| Equation/ Expression | Ordered Pairs | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

## Item Types

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|---|---|---|---|
| Multiselect (2 pts) | Match Table Grid (2 pts) | Drag and Drop (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

## Academic Vocabulary

equation ($y = kx$)*
proportional situation

## Interesting Items

8.5(A) 2023 #39
8.5(A) 2018 #14
8.5(A) 2017 #8

## Role in Concept Development

**Supports**

8.4(B) graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship

**Connection/ Relevance**

Representing linear proportional situations with tables, graphs, and equations is the foundation for students to graph proportional relationships and interpret the unit rate as the slope of the line.

**When to Teach**

Before/Prerequisite to 8.4(B)

**Instructional Implications**

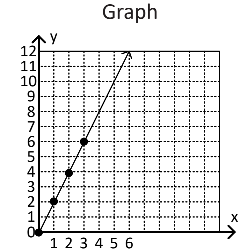
Students represent linear proportional situations with tables, graphs, and equations ($y = kx$). Instruction should include meaningful problems to represent linear proportional situations with rational number constants and coefficients (e.g., in a walkathon, a sponsor will donate $2 per mile the participant walks).

The use of tables organizes data and provides a means for students to develop the process/rule that represents the relationship between independent and dependent quantities (the unit rate, where $k = y/x$ and $x = 1$, remains the same). The use of a process column identifying the rule can support students in representing the data using symbolic notation for equations of the form $y = kx$ (e.g., $y = 2x$).

Table

| Number of Miles Walked | Process | Amount of Donation ($y$) |
|---|---|---|
| 0 | 2(0) | 0 |
| 1 | 2(1) | 2 |
| 2 | 2(2) | 4 |
| 3 | 2(3) | 6 |
| $x$ | 2($x$) | $y$ |

Graph




Graphs allow students to visualize the relationship between the independent and dependent quantities and observe the key components of linear proportional relationships, including the unit rate [e.g., $k = y/x$ when $x = 1$ represents the slope of the line, $y = kx$, and the line contains the origin, (0, 0)]. Students should be able to move fluently between representations (e.g., table, graph, and equation).

**Learning from Mistakes**

Students may make the following mistakes:

- Determining the slope as $x/y$ instead of $y/x$
- Calculating the slope from a table using only the change in $y$ and not taking the change in $x$ into consideration*
- Determining the equation of a function from a proportional situation without a table or graph*

 * Used on STAAR 

# 8.5(E) Supporting

Subcluster: Proportional Reasoning

8.5(E)

**8.5 Proportionality.** The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to:

**(E) solve problems involving direct variation**

## Stimulus

| | | | |
|---|---|---|---|
| Word Problem* | Verbal Description | Chart/Table | Graph |
| Equation/ Expression* | Ordered Pairs | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

## Item Types

| | | | |
|---|---|---|---|
| Multiselect (2 pts) | Match Table Grid (2 pts) | Drag and Drop (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

## Academic Vocabulary

direct variation/varies directly*
equation ($y = kx$)

## Interesting Items

8.5(E) 2022 #40
8.5(E) 2016 #9

## Role in Concept Development

**Supports**

8.4(B) graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship

**Connection/ Relevance**

Being able to solve problems involving direct variation (e.g., $y = kx$) supports students as they graph proportional relationships and interpret the unit rate (e.g., $k$ in the equation, $y = kx$) as the slope of the line.

**When to Teach**

With 8.4(B)

**Instructional Implications**

Students should solve problems involving direct variation (e.g., the number of gallons of gas a car uses varies directly with the number of miles driven, where $y$ represents the number of gallons of gas, $x$ represents the number of miles driven, and $k$ is the unit rate of number of gallons of gas to drive 1 mile) and is written as the equation, $y = kx$, where $k$ is the unit rate and slope, $k = y/x$.

Instruction should include a variety of direct variation problems for students to solve. Students should be familiar with verbal descriptions of direct variation (e.g., the value $y$ varies directly with $x$) and relate this to representations of linear proportional relationships.

Consider the following example: The number of miles represented on a map varies directly with the number of centimeters measured on that map, where the scale is shown as 2 cm = 25 miles. If it is 3.8 cm between two cities on the map, how many miles apart are the two cities? The $y$-values represent the number of miles, the $x$-values represent the number of centimeters, and the unit rate, $k = \frac{\text{number of miles}}{\text{number of centimeters}}$.

**Learning from Mistakes**

Students may make the following mistakes:

- Not relating the constant of proportionality $k = y/x$ to problems involving direct variation*

## TEKS Scaffold

| TEKS | Student Expectation |
|---|---|
| A.3(B) | calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in contents of mathematical and real-world problems (S) |
| 8.4(C) | **8.4 Proportionality.** The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope. The student is expected to: **(C) use data from a table or graph to determine the rate of change or slope and *y*-intercept in mathematical and real-world problems** |
| 7.4(A) | represent constant rates of change in mathematical and real-world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including $d = rt$ (R) |
| 7.4(C) | determine the constant of proportionality ($k = y/x$) within mathematical and real-world problems (S) |
| 6.6(C) | represent a given situation using verbal descriptions, tables, graphs, and equations in the form $y = kx$ or $y = x + b$ (R) |

## Stimulus

| | | | |
|---|---|---|---|
| Word Problem* | Verbal Description* | Chart/Table* | Graph* |
| Equation/ Expression | Ordered Pairs | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

## Item Types

| | | | |
|---|---|---|---|
| Multiselect (2 pts) | Match Table Grid (2 pts) | Drag and Drop (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice* (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor* (1-2 pts) | Multiple Choice* (1 pt) | |

## Academic Vocabulary

function*
rate of change*
slope*, *m*
*x*-intercept
*y*-intercept*

## Interesting Items

8.4(C) 2023 #24
8.4(C) 2023 #32
8.4(C) 2022 #11
8.4(C) 2018 #38
8.4(C) 2018 #38
8.4(C) 2017 #6

## Content Builder (see Appendix for Tree Diagram)

- Use data from a table to determine rate of change or slope and *y*-intercept in mathematical problems
- Use data from a table to determine rate of change or slope and *y*-intercept in real-world problems
- Use data from a graph to determine rate of change or slope and *y*-intercept in mathematical problems
- Use data from a graph to determine rate of change or slope and *y*-intercept in real-world problems

## Instructional Implications

Students should understand that slope is the rate of change. Students should use data from a table or graph to determine the rate of change or slope and the *y*-intercept. For example,

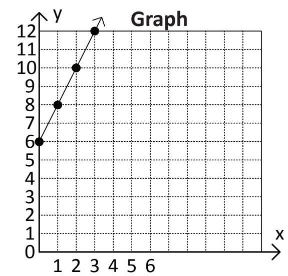
- Data from the table below: $\frac{\text{difference between } y \text{ values}}{\text{difference between } x \text{ values}}$ = slope and the *y*-value when $x = 0$ is the *y*-intercept
- Data from the graph below: $\frac{\text{rise}}{\text{run}}$ = slope and $(0, y)$ is the *y*-intercept

Instruction should include mathematical and real-world problems.

**Table**

| Number of miles walked ($x$) | Process | Amount of Donation ($y$) |
|---|---|---|
| 0 | 2(0) + 6 | 6 |
| 1 | 2(1) + 6 | 8 |
| 2 | 2(2) + 6 | 10 |
| 3 | 2(3) + 6 | 12 |
| $x$ | 2($x$) + 6 | $y$ |

**Graph**




**Data from Table**

Slope: = $\frac{8-6}{1-0} = \frac{2}{1}$

*y*-intercept: 6 since $x$=0

**Data from Graph**

Slope: = $\frac{\text{rise}}{\text{run}} = \frac{2}{1}$

*y*-intercept: (0, 6)

## Learning from Mistakes

Students may make the following mistakes:

- Confusing the *x*-intercept with the *y*-intercept [e.g., *x*-intercept = (0, *y*) and *y*-intercept = (*x*, 0)]*
- Not relating the data from the table (e.g., $\frac{\text{change in } y}{\text{change in } x}$) to the rate of change or slope*
- Thinking the slope is the unit rate ($y/x$) for a non-proportional linear relationship
- Using $\frac{\text{run}}{\text{rise}}$ as the slope when using data from the graph*
- Reversing the slope and dividing the change in *x* by the change in *y* instead of $y/x$
- Having difficulty identifying the constant and/or rate of change/slope for a non-proportional table of values*
- Relying on the visual of the graph to determine the *y*-intercept instead of applying knowledge of (0,*y*)*

# 8.5(I) Readiness

Subcluster: Non-Proportional Reasoning

## TEKS Scaffold

| TEKS | Student Expectation |
|---|---|
| A.2(C) | write linear equations in two variables given a table of values, a graph, and a verbal description (R) |
| 8.5(I) | **8.5 Proportionality.** The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to: **(I) write an equation in the form $y = mx + b$ to model a linear relationship between two quantities using verbal, numerical, tabular, and graphical representations** |
| 7.7(A) | represent linear relationships using verbal descriptions, tables, graphs, and equations that simplify to the form $y = mx + b$ (R) |
| 7.4(A) | represent constant rates of change in mathematical and real-world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including $d = rt$ (R) |
| 6.6(C) | represent a given situation using verbal descriptions, tables, graphs, and equations in the form $y = kx$ or $y = x + b$ (R) |

## Stimulus

| | | | |
|---|---|---|---|
| Word Problem* | Verbal Description | Chart/Table* | Graph* |
| Equation/ Expression* | Ordered Pairs | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

## Item Types

| | | | |
|---|---|---|---|
| Multiselect (2 pts) | Match Table Grid (2 pts) | Drag and Drop* (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

## Content Builder (see Appendix for Tree Diagram)

- Write an equation of the form $y = mx + b$ to model linear relationships between two quantities:
  - verbal
  - numerical
  - tabular
  - graphical

## Instructional Implications

Students write equations in the form $y= mx + b$ to model linear relationships between two quantities using verbal, numerical, tabular, and graphical representations. Instruction should include meaningful situations to represent the linear relationship with rational number constants and coefficients.

This standard distinguishes between proportional and non-proportional functions. Students should recognize that $y = mx + b$ can also be used to represent proportional linear functions when $k = m$ and $b = 0$. Instruction should include the use of mathematical and real-world problems presented as verbal, numerical, tabular, and graphical representations. Students should be encouraged to use precise mathematical language when describing the slope or $y$-intercept of a linear relationship, as opposed to questions such as "what is $m$" or "what is $b$".

In addition, students should be encouraged to describe the meaning of the components of $y = mx + b$ in terms of the problem situation. Students should become proficient in writing equations for linear relationships in formats such as: $y = b + mx$; $mx + b = y$; $b + mx = y$; and $y = mx + b$.

Since positive and negative rational number constants and coefficients should be used, students should be flexible in their methods for writing equations in slope-intercept form and understand that variables other than x or y may be used as defined by the problem situation (e.g., $t = 30m - 50$, where $t$ is the total amount earned and $m$ is the number of magazine subscriptions sold).

## Learning from Mistakes

Students may make the following mistakes:

- Confusing the $x$-intercept with the $y$-intercept
- Not relating the data from the table (e.g., $\frac{\text{change in } y}{\text{change in } x}$) to the rate of change or slope
- Having difficulty identifying the $y$-intercept from a table of values that does not begin the pattern with a zero value as the input*
- Using $\frac{\text{run}}{\text{rise}}$ as the slope when using data from the graph*
- Dividing the change in $x$ by the change in $y$ when determining the slope from coordinates
- Confusing the real-world context of slope/rate of change and the $y$-intercept/constant*

## Academic Vocabulary

constant rate of change
slope
$y$-intercept
$y = mx + b$

## Interesting Items

8.5(I) 2023 #20
8.5(I) 2021 #10
8.5(I) 2021 #21
8.5(I) 2018 #28
8.5(I) 2017 #19

8.5(B) **8.5 Proportionality.** The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to:

**(B) represent linear non-proportional situations with tables, graphs, and equations in the form of *y* = *mx* + *b*, where *b* ≠ 0**

## Stimulus

| | | | |
|---|---|---|---|
| Word Problem* | Verbal Description | Chart/Table* | Graph* |
| Equation/ Expression | Ordered Pairs | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

## Item Types

| | | | |
|---|---|---|---|
| Multiselect* (2 pts) | Match Table Grid (2 pts) | Drag and Drop (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

## Academic Vocabulary

non-proportional situation
slope
*y*-intercept
*y* = *mx* + *b*

## Interesting Items

8.5(B) 2023 #22

## Role in Concept Development

**Supports**

8.4(C) use data from a table or graph to determine the rate of change or slope and *y*-intercept in mathematical and real-world problems

**Connection/ Relevance**

Representing linear non-proportional situations with tables, graphs, and equations is the foundation for students to use data from a table or graph to determine the slope and *y*-intercept of linear relationships in mathematical and real-world problems.

**When to Teach**

With 8.4(C)

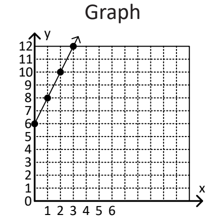
**Instructional Implications**

Students should represent linear non-proportional situations with tables, graphs, and equations (*y* = *mx* + *b*). Instruction should include meaningful problems to represent linear non-proportional situations with rational number constants and coefficients (e.g., in a walkathon, a sponsor will donate $6 and an additional $2 per mile the participant walks). It is important students develop an understanding that each representation is a different way to communicate the relationship between the quantities *x* and *y*. The use of tables organizes data and provides a means for students to develop the process/rule that represents the relationship between independent and dependent quantities (e.g., the slope, *m*, is the rate comparing the change in *y*-values to the change in *x*-values, $\frac{y_2 - y_1}{x_2 - x_1}$, and the *y*-intercept, *b*, is represented by the *y*-value in the ordered pair (0, *y*), where *b* ≠ 0). The use of a process column identifying the rule can support students in representing the data using symbolic notation for equations of the form *y* = *mx* + *b*, *b* ≠ 0 (e.g., *y* = 2*x* + 6).

Table

| Number of Miles Walked | Process | Amount of Donation (*y*) |
|---|---|---|
| 0 | 2(0) + 6 | 6 |
| 1 | 2(1) + 6 | 8 |
| 2 | 2(2) + 6 | 10 |
| 3 | 2(3) + 6 | 12 |
| *x* | 2(*x*) + 6 | *y* |

Graph




Graphs allow students to visualize the relationship between the independent and dependent quantities and observe key components of linear non-proportional relationships. Linear relationships have a constant rate of change (*m* in the equation *y* = *mx* + *b*) and a *y*-intercept [the point (0, *b*)]. Students should understand the meaning of *m* and *b* within *y* = *mx* + *b* in terms of the context of the problem situation (e.g., *y* = 2*x* + 6; the *y*-intercept represents the $6 donation and the *m* represents the $2 increase in donation for each mile that is walked). Students should be able to move fluently from one representation (e.g., table, graph, and equation) to the next.

(continued)


* Used on STAAR

## Role in Concept Development (continued)

**Learning from Mistakes**

Students may make the following mistakes:

- Not recognizing $y = 6 + 3x$ as equivalent to $y = 3x + 6$
- Transposing the slope ($m$) and $y$-intercept ($b$) within the algebraic representation of a problem situation
- Not considering the constant ($b$) in the problem and interpreting the situation as proportional
- Calculating the slope from a table using only the change in $y$ and not taking the change in $x$ into consideration
- Reversing the slope and the $y$-intercept when moving between representations
- Determining the rate of change as $x/y$ instead of $y/x$
- Confusing the $x$-intercept with the $y$-intercept

8.9(A)

**8.9 Expressions, equations, and relationships.** The student applies mathematical process standards to use multiple representations to develop foundational concepts of simultaneous linear equations. The student is expected to:

**(A) identify and verify the values of *x* and *y* that simultaneously satisfy two linear equations in the form *y* = *mx* + *b* from the intersections of the graphed equations**

## Stimulus

| | | | |
|---|---|---|---|
| Word Problem* | Verbal Description* | Chart/Table | Graph* |
| Equation/ Expression | Ordered Pairs* | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

## Item Types

| | | | |
|---|---|---|---|
| Multiselect (2 pts) | Match Table Grid (2 pts) | Drag and Drop (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

## Academic Vocabulary

intersecting lines
point of intersection

## Interesting Items

8.9(A) 2016 #50

## Role in Concept Development

**Supports**

A.5(C) solve systems of two linear equations with two variables for mathematical and real-world problems

**Connection/ Relevance**

Being able to identify and verify the values of *x* and *y* where two lines intersect provides the foundational understanding for systems of linear equations.

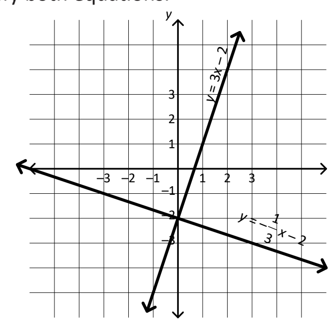
**When to Teach**

After 8.5(B)/8.5(H)

**Instructional Implications**

Students should identify and verify whether the coordinates of the intersection of graphed linear equations simultaneously satisfy both equations.

For example, graph the two equations: $y = 3x - 2$ and $y = -\frac{1}{3}x - 2$; (0, -2) are the coordinates of the ordered pair where the two lines intersect. To verify the *x*- and *y*-values satisfy the two equations, the values are substituted in each equation to determine if the simplified equation results in two true statements [e.g., (0, -2) is the point of intersection because $0 - 2 = 3(-2) - 2$ and $-2 = \frac{1}{3}(0) - 2$ are both true statements].




It is important that instruction includes equations where the lines intersect at exactly one point [(0, 3) for $y = 2x + 3$ and $y = 0.4x + 3$]. Students should be able to explain the meaning of the point that simultaneously satisfies two linear equations in terms of the given situation.

**Learning from Mistakes**

Students may make the following mistakes:

- Transposing the ordered pairs of the point of intersection as (*y*, *x*) instead of (*x*, *y*)
- Incorrectly describing the meaning of the point that simultaneously satisfies both linear equations*

8.5(F)

**8.5 Proportionality.** The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to:

**(F) distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form $y = kx$ or $y = mx + b$, where $b \neq 0$**

## Stimulus

| | | | |
|---|---|---|---|
| Word Problem | Verbal Description | Chart/Table | Graph* |
| Equation/ Expression | Ordered Pairs | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

## Item Types

| | | | |
|---|---|---|---|
| Multiselect (2 pts) | Match Table Grid (2 pts) | Drag and Drop (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

## Academic Vocabulary

non-proportional*
proportional*

## Interesting Items

8.5(F) 2023 #7
8.5(F) 2021 #37
8.5(F) 2016 #2

## Role in Concept Development

**Supports**

8.5(I) write an equation in the form $y = mx + b$ to model a linear relationship between two quantities using verbal, numerical, tabular, and graphical representations

## Role in Concept Development (continued)

**Connection/ Relevance**

Being able to distinguish between proportional and non-proportional situations is the foundation for students to write equations in the form $y = mx + b$ to model linear relationships.

**When to Teach**

Before/Prerequisite to 8.5(I)

**Instructional Implications**

Students should distinguish between proportional and non-proportional situations with tables, graphs, and equations (e.g., $y = kx$; $y = mx + b$, $b \neq 0$; $y = mx + b$).

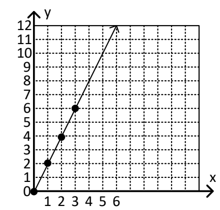
Instruction should include meaningful problems to represent both proportional situations (e.g., in a walkathon, a sponsor will donate $2 per mile the participant walks) and non-proportional situations (e.g., in a walkathon, a sponsor will donate $6 and an additional $2 per mile the participant walks) through the use of tables, graphs, and equations (e.g., $y = 2x$ or $y = 2x + 6$).

Side by side comparisons of the two situations provides a means for students to distinguish between proportional and non-proportional linear situations (e.g., the proportional situation is a line that contains the origin and the slope is the unit rate ($k = y/x$); the non-proportional situation is a line that has a $y$-intercept of $(0, b)$, where $b \neq 0$ and the slope is the rate ($\frac{\text{change in } y \text{ values}}{\text{change in } x \text{ values}}$).

Proportional Table

| Number of Miles Walked | Process | Amount of Donation ($y$) |
|---|---|---|
| 0 | 2(0) | 0 |
| 1 | 2(1) | 2 |
| 2 | 2(2) | 4 |
| 3 | 2(3) | 6 |
| $x$ | 2($x$) | $y$ |

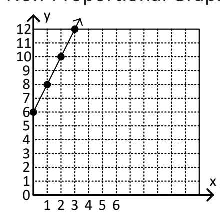
Proportional Graph



Non-Proportional Table

| Number of Miles Walked | Process | Amount of Donation ($y$) |
|---|---|---|
| 0 | 2(0) + 6 | 6 |
| 1 | 2(1) + 6 | 8 |
| 2 | 2(2) + 6 | 10 |
| 3 | 2(3) + 6 | 12 |
| $x$ | 2($x$) + 6 | $y$ |

Non-Proportional Graph



**Learning from Mistakes**

Students may make the following mistakes:
- Assuming all straight lines on a graph represent a proportional relationship
- Assuming that if a table starts at a value, other than zero, then it is non-proportional

8.5(H)

**8.5 Proportionality.** The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to:

**(H) identify examples of proportional and non-proportional functions that arise from mathematical and real-world problems**

## Stimulus

| | | | |
|---|---|---|---|
| Word Problem | Verbal Description* | Chart/Table | Graph |
| Equation/ Expression | Ordered Pairs | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

## Item Types

| | | | |
|---|---|---|---|
| Multiselect (2 pts) | Match Table Grid (2 pts) | Drag and Drop (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

## Academic Vocabulary

non-proportional
proportional*

## Interesting Items

8.5(H) 2024 #26
8.5(H) 2022 #18
8.5(H) 2016 #20

## Role in Concept Development

**Supports**

8.5(I) write an equation in the form $y = mx + b$ to model a linear relationship between two quantities using verbal, numerical, tabular, and graphical representations

**Connection/ Relevance**

Being able to identify examples of proportional and non-proportional functions from mathematical and real-world problems is the foundation for students to write equations in the form $y = mx + b$ to model linear relationships.

**When to Teach**

With 8.5(I)

**Instructional Implications**

Students should identify proportional and non-proportional functions from mathematical and real-world problems. Instruction should include a variety of problems involving proportional situations (e.g., an insurance agent earns a commission of 21% of the value of the insurance policies she sells) and non-proportional situations (e.g., an insurance agent earns $1500 a month base pay and earns a commission of 9% of the value of the insurance policies she sells).

Students can use various methods, such as tables, graphs, and equations, to determine whether mathematical and/or real-world problems are proportional.

**Learning from Mistakes**

Students may make the following mistakes:

- Assuming all straight lines on a graph represent a proportional relationship
- Assuming that if a table starts at a value, other than zero, then it is non-proportional
- Having difficulty relating proportional/non-proportional relationships to geometric concepts*

## TEKS Scaffold

| TEKS | Student Expectation |
|---|---|
| A.12(A) | decide whether relations represented verbally, tabularly, graphically, and symbolically define a function (S) |
| 8.5(G) | **8.5 Proportionality.** The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to:<br>**(G) identify functions using sets of ordered pairs, tables, mappings, and graphs** |

## Stimulus

| | | | |
|---|---|---|---|
| Word Problem | Verbal Description* | Chart/Table* | Graph* |
| Equation/ Expression | Ordered Pairs* | Diagram/Image* | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

## Item Types

| | | | |
|---|---|---|---|
| Multiselect* (2 pts) | Match Table Grid (2 pts) | Drag and Drop (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

## Content Builder (see Appendix for Tree Diagram)

- Identify functions using:
  - sets of ordered pairs
  - tables
  - mappings
  - graphs

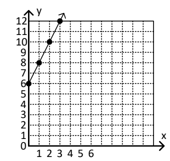
## Instructional Implications

Students identify functions using sets of ordered pairs [e.g., identify function when given (0,1), (2,3), (4,5) or (0,1), (2,3), (2,5)], tables, mappings, and graphs. Instruction should include a variety of situations for students to relate to functions (e.g., Is it possible for an individual to have more than one birthdate?; Is it possible for more than one individual to share the same birthday?). The vertical line test may also be used to determine if a graph represents a function (i.e. if a vertical line passes through two or more points on a graph, the graph does not represent a function).

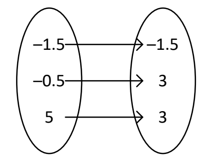
Table

| ($x$) | ($y$) |
|---|---|
| 0 | 6 |
| 1 | 8 |
| 2 | 10 |
| 3 | 12 |

Graph



Mapping




Instruction should include opportunities for students to formally define a function as well as distinguish between functions and relations. This standard does not require the use of function notation [e.g., $f(x)$].

## Learning from Mistakes

Students may make the following mistakes:

- Thinking two $x$-values mapped to the same $y$-value is not a function*
- Thinking only linear data can represent a function
- Confusing the concept of functions with “repeating values” or “repeating arrows” in a given stimulus*
- Using a horizontal line test instead of a vertical line test to determine functions*
- Not identifying piecewise graphs as a function*

## Academic Vocabulary

function*
$y$ as a function of $x$*

## Interesting Items

8.5(G) 2024 #3
8.5(G) 2022 #36
8.5(G) 2019 #5
8.5(G) 2016 #28