TEKS Cluster: Real Number Relationships

8.2 **Number and operations.** The student applies mathematical process standards to represent and use real numbers in a variety of forms.

Representation of Real Numbers

Supporting Standards

8.2(A) extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of real numbers

8.2(C) convert between standard decimal notation and scientific notation

Magnitude of Real Numbers

Readiness Standards

8.2(D) order a set of real numbers arising from mathematical and real-world contexts

Supporting Standards

8.2(B) approximate the value of an irrational number, including π and square roots of numbers less than 225, and locate that rational number approximation on a number line

8.2(A) **8.2 Number and operations.** The student applies mathematical process standards to represent and use real numbers in a variety of forms. The student is expected to:

(A) extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of real numbers

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Ordered Pairs	Diagram/Image*	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

π (pi)
integer
irrational number*
rational number*
real number*
square root
whole number*

Interesting Items

8.2(A) 2019 #15
8.2(A) 2018 #15

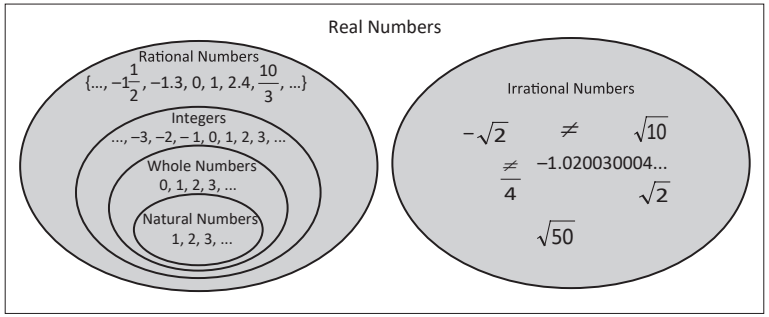
Role in Concept Development

Supports
8.2(D) order a set of real numbers arising from mathematical and real-world contexts

Connection/ Relevance
Describing the relationship between a set of real numbers allows students to have a better understanding of the various representations in order to compare the values in mathematical and real-world contexts.

When to Teach
Before/Prerequisite to 8.2(D)

Instructional Implications
Students use a visual representation (e.g., Venn diagram) to describe relationships between sets of real numbers. Instruction extends students' previous knowledge of sets and subsets of rational numbers (e.g., natural numbers are a subset of whole numbers and integers, integers are subsets of rational numbers, etc.). Subsets of real numbers include counting (or natural) numbers, whole numbers, integers, rational numbers, and irrational numbers.



The set of rational numbers and the set of irrational numbers form the set of real numbers.

Learning from Mistakes
Students may make the following mistakes:
- Confusing the placement of numbers within the Venn diagram as being exclusive to one set rather than realizing that certain sets are subsets of other sets*
- Having difficulty identifying the difference between rational and irrational numbers*
- Thinking that any negative number is an integer
- Thinking any number with a radical is an irrational number

8.2(C) Supporting

8.2(C) **8.2 Number and operations.** The student applies mathematical process standards to represent and use real numbers in a variety of forms. The student is expected to:

(C) convert between standard decimal notation and scientific notation

Stimulus

Word Problem*	Verbal Description	Chart/Table	Graph
Equation/ Expression*	Ordered Pairs	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

scientific notation*

Interesting Items

8.2(C) 2022 #9
8.2(C) 2021#22

Role in Concept Development

Supports

8.2(A) extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of real numbers

Connection/ Relevance

Converting between standard decimal notation and scientific notation builds on composing and decomposing numbers based on place value in order to represent and use real numbers in a variety of forms.

When to Teach

After 8.2(A)

Instructional Implications

Students convert between:

- Standard decimal notation – standard notation where numbers are written without exponents using the base ten system; 34.01
- Scientific notation – a mathematical process used to represent very large or very small numbers where the number is written as a decimal with exactly one non-zero digit (positive or negative) to the left of the decimal point multiplied by a power of 10, positive or negative; 3.01×10^{-6} or -1.25×10^{4}

Students need a variety of problems where they convert from standard decimal notation to scientific notation and vice versa. Instruction should include real-world situations where scientific notation is used (e.g., an approximate population of the world is 7 billion or 7×10^{9}). It is important for students to understand that the two forms represent equivalent values.

Students need to understand that a number becomes 10 times bigger or smaller when multiplying by a power of 10 rather than using the generalization of "moving the decimal". Instruction should include the use of a calculator when converting between standard decimal and scientific notation. Students should understand the meaning of E in relationship to 10^{x}.

Learning from Mistakes

Students may make the following mistakes:

- Assuming that the exponent represents the number of zeros in the standard decimal form of the number*
- Disregarding the magnitude of the negative and positive exponent when representing the value of a number (e.g., $0.034 = 3 \times 10^{2}$ instead of 3×10^{-2})*

8.2(D) Readiness

TEKS Scaffold

TEKS	Student Expectation
8.2(D)	**8.2 Number and operations.** The student applies mathematical process standards to represent and use real numbers in a variety of forms. The student is expected to: **(D) order a set of real numbers arising from mathematical and real-world contexts**
8.2(B)	approximate the value of an irrational number, including π and square roots of numbers less than 225, and locate that rational number approximation on a number line (S)
6.2(D)	order a set of rational numbers arising from mathematical and real-world contexts (R)
6.2(C)	locate, compare, and order integers and rational numbers using a number line (S)

Stimulus

Word Problem*	Verbal Description	Chart/Table*	Graph
Equation/ Expression*	Ordered Pairs	Diagram/Image	Number Line*
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures*

Item Types

Multiselect* (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot* (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Content Builder (see Appendix for Tree Diagram)

- Order a set of real numbers arising from mathematical context
- Order a set of real numbers arising from real-world context

Instructional Implications

In conjunction with 8.2(B), students use number lines to order a set of real numbers arising from mathematical (e.g., π, square roots) and real-world contexts (e.g., newspaper advertisements, stock market values, temperatures). Students should compare/order a mixture of real number representations (e.g., order the following real numbers from least to greatest: $5\frac{1}{2}$, $2\frac{1}{4}$, √5, 5.25, 5%, π).

Real numbers are inclusive of rational numbers (i.e. integers, percents, and positive/negative fractions and decimals) and irrational numbers. Instruction should include using multiple comparison symbols to represent the magnitude of given real numbers (e.g., $65\% < \frac{2}{3} < 0.7$).

Learning from Mistakes

Students may make the following mistakes:

- Disregarding the sign of negative integers when ordering non-positive numbers
- Comparing the number of digits instead of applying their understanding of place value to determine the value of decimals (e.g., 0.451 is greater than 0.98 because it has more digits)
- Not understanding that 0.7 is equivalent to 0.70
- Assuming repeating decimals have more value than non-repeating values since they have more digits (e.g., $0.\overline{27} > .274$)*
- Thinking the square root of a number means dividing the number by 2*
- Viewing 5 and √5 as equivalent values
- Not understanding the context of problems to order real numbers correctly (e.g., when ordering time from fastest to slowest, students may want to order from greatest to least)
- Confusing the use of two comparison symbols to communicate “at least” or “no more than” (e.g., $4.5 > x > 3\frac{1}{2}$)*
- Disregarding the percentage symbol when ordering values including decimals*

Academic Vocabulary

> (greater than)
< (less than)*
= (equal to)
greatest to least*
least to greatest*

Interesting Items

8.2(D) 2023 #30
8.2(D) 2021 #33
8.2(D) 2017 #29
8.2(D) 2016 #37

8.2(B) Supporting

8.2(B) **8.2 Number and operations.** The student applies mathematical process standards to represent and use real numbers in a variety of forms. The student is expected to:

(B) approximate the value of an irrational number, including π and square roots of numbers less than 225, and locate that rational number approximation on a number line

Stimulus

Word Problem*	Verbal Description	Chart/Table	Graph
Equation/ Expression	Ordered Pairs	Diagram/Image	Number Line*
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

π (pi)
√ (square root)*

Interesting Items

8.2(B) 2023 #9
8.2(B) 2021 #39
8.2(B) 2017 #31

Role in Concept Development

Supports
8.2(D) order a set of real numbers arising from mathematical and real-world contexts

Connection/ Relevance
This standard describes the mathematical relationship found in irrational numbers. This relationship supports students in approximating the value of an irrational number in order to effectively order a set of real numbers.

When to Teach
With 8.2(D)

Instructional Implications
This standard builds on students' understanding of placing rational numbers on a number line. This standard applies that knowledge to approximating the value of an irrational number and locating that approximation on a number line.

Instruction should have students approximate the value of several irrational numbers (e.g., π, √2, √50, -1.232232223…, etc.). As students approximate the value of irrational numbers, they should locate these approximated values on a number line.

Students should be able to approximate the square root of a given rational number (e.g., given a square with an area of 51.2 square inches, what is the approximate length of one side?). A calculator should be used to determine the approximate value of square roots that represent irrational numbers.

Learning from Mistakes
Students may make the following mistakes:
- Ignoring the radical of a number and treating it like a whole number (e.g., √9 as 9)
- Dividing a number by two to determine the square root of a number
- Locating negative rational number approximations incorrectly