TEKS Cluster: Geometry

4.6 **Geometry and measurement.** The student applies mathematical process standards to analyze geometric attributes in order to develop generalizations about their properties.

4.7 **Geometry and measurement.** The student applies mathematical process standards to solve problems involving angles less than or equal to 180 degrees.

Two-Dimensional

Readiness Standards

4.6(D) classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size

Supporting Standards

4.6(A) identify points, lines, line segments, rays, angles, and perpendicular and parallel lines

4.6(B) identify and draw one or more lines of symmetry, if they exist, for a two-dimensional figure

4.6(C) apply knowledge of right angles to identify acute, right, and obtuse triangles

Angle Measurements

Readiness Standards

4.7(C) determine the approximate measures of angles in degrees to the nearest whole number using a protractor

Supporting Standards

4.7(D) draw an angle with a given measure

4.7(E) determine the measure of an unknown angle formed by two non-overlapping adjacent angles given one or both angle measures

Non-tested Standards

4.7(A) Illustrate the measure of an angle as the part of a circle whose center is at the vertex of the angle that is "cut out" by the rays of the angle; angle measures are limited to whole numbers

4.7(B) Illustrate degrees as the units used to measure an angle, where 1/360 of any circle is 1 degree and an angle that "cuts" *n*/360 out of any circle whose center is at the angle's vertex has a measure of *n* degrees; angle measures are limited to whole numbers

4.6(D) Readiness

TEKS Scaffold

TEKS	Student Expectation
5.5(A)	classify two-dimensional figures in a hierarchy of sets and subsets using graphic organizers based on their attributes and properties (R)
4.6(D)	**4.6 Geometry and measurement.** The student applies mathematical process standards to analyze geometric attributes in order to develop generalizations about their properties. The student is expected to: **(D) classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size**
3.6(A)	classify and sort two- and three-dimensional figures, including cones, cylinders, spheres, triangular and rectangular prisms, and cubes, based on attributes using formal geometric language (R)
2.8(C)	classify and sort polygons with 12 or fewer sides according to attributes, including identifying the number of sides and number of vertices (R)

Stimulus

Word Problem*	Verbal Description*	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures*

Item Types

Multiselect* (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Content Builder (see Appendix for Tree Diagram)

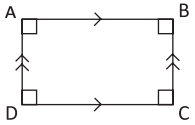
- Classify two-dimensional figures that have a presence or absence of:
  - parallel lines
  - perpendicular lines
  - angles of specified size

Instructional Implications

Students compare/contrast the attributes (e.g., number of vertices, sides, angles) and presence or absence of properties (e.g., parallel lines, perpendicular lines, number/type/size of angles) of various two-dimensional figures. According to 4.6, students are able to better classify two-dimensional figures by developing generalizations about their properties, such as:

- All parallelograms can be called quadrilaterals because they have four sides and four angles, but cannot all be called rectangles because not all parallelograms have four right angles
- A trapezoid can be called a quadrilateral because it has four sides and four angles, but cannot be called a parallelogram because it only has one set of parallel sides

Students should understand the following geometric symbols:



- Angles A, B, C, and D are right angles
- $\overline{AB} \perp \overline{BC}$
  Line segment AB is perpendicular to line segment BC
- $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$
  The chevrons indicate that line segment AB is parallel to line segment DC, and line segments BC and AD are also parallel

Learning from Mistakes

Students may make the following mistakes:

- When identifying parallel lines, not understanding that the lines do not have to be equal in length nor be shown directly above/below each other

- Struggling with visualizing geometric figures with given attributes when pictures or images are not provided*
- Identifying polygons by the way they look instead of by attributes (e.g., not being able to identify a right trapezoid as a type of trapezoid)*
- Not recognizing a quadrilateral as having more than one classification (e.g., a square is also a polygon, rectangle, parallelogram, quadrilateral, rhombus)*

Academic Vocabulary

angle*
parallel*
perpendicular*
right angle*

Interesting Items

English

4.6(D) 2023 #9
4.6(D) 2022 #17
4.6(D) 2018 #19
4.6(D) 2017 #5
4.6(D) 2017 #20

Spanish

4.6(D) 2022 #17
4.6(D) 2018 #19
4.6(D) 2017 #5
4.6(D) 2017 #20

4.6(A) **4.6 Geometry and measurement.** The student applies mathematical process standards to analyze geometric attributes in order to develop generalizations about their properties. The student is expected to:

(A) identify points, lines, line segments, rays, angles, and perpendicular and parallel lines

Stimulus

Word Problem	Verbal Description*	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures*

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop* (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

acute angle*
angle*
intersecting lines*
line segment*
obtuse angle*
parallel*
perpendicular*
point
ray
right angle*

Interesting Items

English
4.6(A) 2022 #4
4.6(A) 2016 #40

Spanish
4.6(A) 2022 #4
4.6(A) 2016 #40

Role in Concept Development

Supports
4.6(D) classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size

Connection/ Relevance
In order for students to appropriately classify two-dimensional figures, it is critical to identify the attributes of points, lines, line segments, rays, angles, and perpendicular and parallel lines.

When to Teach
Before/Prerequisite to 4.6(D)

Instructional Implications
In conjunction with 4.6(D), students need to understand the geometric attributes outlined in this standard in order to classify two-dimensional figures. According to 4.6, students need to be able to identify and analyze the attributes of the various geometric terms.

Term	Example	Geometric Properties
Point		Specifies location; does not have length, width, or depth
Line		Has only one dimension of length; does not have width or depth; connects two points and continues on in both directions
Line Segment		Has only one dimension of length; does not have width or depth; two points (called endpoints) and all points between
Ray		Contains a point (endpoint) and extends one direction
Angle		A figure formed by two rays that share the same endpoint (vertex)
Perpendicular Lines		Two lines that intersect forming a 90° (right) angle
Parallel Lines		Two lines that never intersect

Learning from Mistakes
Students may make the following mistakes:
- Overgeneralizing that all intersecting lines are perpendicular*
- Not realizing that parallel lines can be different sizes and need not be directly next to each other
- Not differentiating between an angle and a ray (e.g., seeing all rays as angles and/or vice versa)
- Not differentiating between a line and a line segment*

4.6(B) **4.6 Geometry and measurement.** The student applies mathematical process standards to analyze geometric attributes in order to develop generalizations about their properties. The student is expected to:

(B) identify and draw one or more lines of symmetry, if they exist, for a two-dimensional figure

Stimulus

Word Problem	Verbal Description*	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures*

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

congruent
line of symmetry*
symmetry*

Interesting Items

English
4.6(B) 2023 #1
4.6(B) 2017 #17
4.6(B) 2016 #11

Spanish
4.6(B) 2017 #17
4.6(B) 2016 #11

Role in Concept Development

Supports

- 6.8(D) determine solutions for problems involving the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers
- 6.8(B) model area formulas for parallelograms, trapezoids, and triangles by decomposing and rearranging parts of these shapes

Connection/ Relevance

Identifying lines of symmetry for two-dimensional figures supports students in their future study of decomposing and rearranging their parts to model various formulas (e.g., a rectangle has a diagonal line which yields two congruent triangles; therefore, the formula for a triangle is half of rectangle; $A = \frac{1}{2}bh$).

When to Teach

Before/Prerequisite to 6.8(B) and 6.8(D)

Instructional Implications

Students must identify and physically draw lines of symmetry. The standard limits the figures to two-dimensional; however, instruction should include regular and irregular two-dimensional shapes. Encourage students to identify and draw more than one line of symmetry, should more than one exist.

According to 4.6, students need to develop generalizations about lines of symmetry (e.g., lines of symmetry divide a figure into two congruent parts; corresponding points on each half are equidistant from the line of symmetry).

Learning from Mistakes

Students may make the following mistakes:

- Not realizing that some figures have more than one line of symmetry; especially diagonal lines of symmetry*
- Overgeneralizing that all regular figures have lines of symmetry and all irregular figures have no lines of symmetry
- Incorrectly believing that because rectangles have diagonals, those diagonals create lines of symmetry

4.6(C) Supporting

4.6(C) **4.6 Geometry and measurement.** The student applies mathematical process standards to analyze geometric attributes in order to develop generalizations about their properties. The student is expected to:

(C) apply knowledge of right angles to identify acute, right, and obtuse triangles

Stimulus

Word Problem	Verbal Description*	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures*

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice* (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

acute angle*
angle*
obtuse angle*
right angle

Interesting Items

English	*Spanish*
4.6(C) 2024 #31	4.6(C) 2021 #17
4.6(C) 2023 #21	4.6(C) 2017 #29
4.6(C) 2021 #17	4.6(C) 2016 #20

Role in Concept Development

Supports

- 4.6(D) classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size
- 4.7(C) determine the approximate measures of angels in degrees to the nearest whole number using a protractor

Connection/ Relevance

Identifying the types of angles (acute, obtuse, and right) supports the classification of two-dimensional figures. Knowing the measurements of the different angle types is important when using a protractor to measure angles.

When to Teach

Before/Prerequisite to 4.6(D) and 4.7(C)

Instructional Implications

In 4.6(A), students identified perpendicular lines and described how the angles formed from perpendicular lines are 90°, right angles. As students begin measuring angles [see 4.7 (C)/(D)], they can begin to classify angles into three categories: acute (less than 90°), right (exactly 90°), and obtuse (greater than 90°).

This knowledge of angle classification can then define various triangles:

- Acute triangle: all angles measure less than 90°
- Right triangle: one angle measures 90°
- Obtuse triangle: one angle measures more than 90°

Students should realize that all triangles are named according to their largest angle.

Learning from Mistakes

Students may make the following mistakes:

- Misreading the angles on a triangle and selecting the wrong angle measure to name the triangle*
- Misidentifying angles on a rotated polygon* (e.g., )

4.7(C) Readiness

TEKS Scaffold

TEKS	Student Expectation
5.5(A)	classify two-dimensional figures in a hierarchy of sets and subsets using graphic organizers based on their attributes and properties (R)
4.7(D)	draw an angle with a given measure (S)
4.7(C)	**4.7 Geometry and measurement.** The student applies mathematical process standards to solve problems involving angles less than or equal to 180 degrees. The student is expected to: **(C) determine the approximate measures of angles in degrees to the nearest whole number using a protractor**
4.7(B)	illustrate degrees as the units used to measure an angle, where 1/360 of any circle is 1 degree and an angle that "cuts" *n*/360 out of any circle whose center is at the angle's vertex has a measure of *n* degrees; angle measures are limited to whole numbers
4.7(A)	illustrate the measure of an angle as the part of a circle whose center is at the vertex of the angle that is "cut out" by the rays of the angle; angle measures are limited to whole numbers

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image*	Number Line
Base Ten Blocks	Measurement Tool*	Formula	Geometric Figures

Item Types

Multiselect* (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Content Builder (see Appendix for Tree Diagram)

- Determine the approximate measure of angles in degrees to the nearest whole number using a protractor

Instructional Implications

Just like a ruler can measure length in inches or a clock can measure time in minutes/hours, a protractor can measure angles in degrees. Instruction should identify the different components of a protractor (inner scale, outer scale, center mark, zero edge). In conjunction with 4.6(C), students should first identify a given angle as acute or obtuse and then estimate an angle measurement prior to actually measuring. Instruction should model how to place the center of the protractor on the vertex of the angle and align the zero edge of the protractor with one of the rays of the angle. The point where the other ray intersects the protractor determines the measurement of the angle. Depending whether the given angle was previously identified as an acute or obtuse angle determines whether to record the inner (acute) or outer (obtuse) angle measure reflected on the protractor.

Students should notice how the protractor represents a half of a circle or 180°. Just as a ruler only measures lengths up to 12 inches, a protractor only allows the measurement of angles of 180°or less. The composing (e.g., an image of a 260° angle would have to be decomposed into two separate angles with each angle measurement added together) or decomposing (e.g., the measure of the inner angle could be subtracted from 360°) of angles would have to be applied to determine the measurement of an angle measuring more than 180°.

Instruction should also include the use of images of a protractor and a given angle to determine the measure of the angle. Examples may or may not align to the zero edge of the protractor (e.g., an image of a protractor reflecting one ray aligning with the 50° unit of measure and the other ray aligning with the 130° unit of measure; 130° – 50° = 80°; angle measures 80°).

Learning from Mistakes

Students may make the following mistakes:

- Reading the wrong scale of the protractor (if students are not identifying whether a measurable angle is acute or obtuse)*
- Having difficulty manipulating a protractor to align rays that may be rotated
- Thinking the length of the rays relate to the size of the angle measure
- When given an image of a protractor that does not align at the zero edge, looking at where a ray intersects the protractor rather than calculating the angle measure*

Academic Vocabulary

angle*
angle vertex
center mark
degree*
inner scale
outer scale
protractor*
ray
zero edge

Interesting Items

English	*Spanish*
4.7(C) 2024 #13	4.7(C) 2016 #23
4.7(C) 2016 #23	4.7(C) 2016 #46

4.7(D) **4.7 Geometry and measurement.** The student applies mathematical process standards to solve problems involving angles less than or equal to 180 degrees. The student is expected to:

(D) draw an angle with a given measure

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool*	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice* (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

angle*
angle vertex
center mark
degree*
inner scale
outer scale
protractor*
ray*
zero edge

Interesting Items

English
4.7(D) 2024 #18
4.7(D) 2021 #12

Spanish
4.7(D) 2024 #18
4.7(D) 2021 #12
4.7(D) 2019 #6
4.7(D) 2015 #17

Role in Concept Development

Supports
4.7(C) determine the approximate measures of angles in degrees to the nearest whole number using a protractor

Connection/ Relevance
Drawing angles of a given measure builds the student's efficiency in the use of a protractor.

When to Teach
With 4.7(C)

Instructional Implications
In conjunction with 4.7(C), instruction should extend from measuring angles using a protractor to drawing the angle. Instruction should begin with students identifying whether the given angle measurement is acute or obtuse. Students should move on to drawing a ray by:
- Aligning the center of the protractor to what becomes the vertex of the angle
- Aligning the zero edge of the protractor to the drawn edge
- Marking the given angle measurement (remind students to refer to the appropriate unit marks for acute/obtuse angles)
- Drawing a ray from the center of the angle to the indicated mark

Learning from Mistakes
Students may make the following mistakes:
- Misidentifying the type of the angle to be drawn (e.g., when asked to draw a 35 degree angle, not being able to visualize an acute angle)
- Misaligning the zero edge of the protractor
- Using the wrong scale of the protractor (if students are not identifying whether a measurable angle is acute or obtuse)

4.7(E) Supporting

4.7(E)

4.7 Geometry and measurement. The student applies mathematical process standards to solve problems involving angles less than or equal to 180 degrees. The student is expected to:

(E) determine the measure of an unknown angle formed by two non-overlapping adjacent angles given one or both angle measures

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image*	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

angle*
adjacent angle
degree

Interesting Items

English
4.7(E) 2016 #3
4.7(E) 2015 #18

Spanish
4.7(E) 2018 #7
4.7(E) 2016 #3
4.7(E) 2015 #18

Role in Concept Development

Supports
4.7(C) determine the approximate measures of angles in degrees to the nearest whole number using a protractor

Connection/ Relevance
As students have to determine the approximate measure of angles greater than 180^0, determining the measure of adjacent angles may be necessary when using a protractor.

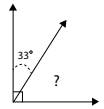
When to Teach
After 4.7(C)

Instructional Implications
Instruction should define and provide examples and non-examples of non-overlapping adjacent angles. Adjacent angles are to be defined as two angles that do not overlap but share a common ray and a common point (vertex). Students should begin to visualize how two adjacent angles yield a larger angle.

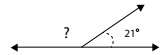
As students begin to determine the measure of unknown angles, be sure to vary the context of the problems. This understanding begins to lay the foundation for supplementary and complementary angle measurements.

For example:

- Given two adjacent angles, determine the measurement of the larger angle
- Given the measurement of the larger angle and one of the adjacent angles, determine the measurement of the other adjacent angle
- Given that the larger angle is a right angle and the measure of one of the adjacent angles, determine the measure of the other adjacent angle
- Given that the larger angle represents half of a circle and the measure of one of the adjacent angles, determine the measure of the other adjacent angle



Complementary



Supplementary

Learning from Mistakes
Students may make the following mistakes:

- Not realizing that a right angle (or the square symbol for a right angle) measures 90 degrees*
- Not realizing that a straight angle (line) measures 180 degrees

4.7 Geometry and measurement. The student applies mathematical process standards to solve problems involving angles less than or equal to 180 degrees. The student is expected to:

4.7(A) **(A) illustrate the measure of an angle as the part of a circle whose center is at the vertex of the angle that is "cut out" by the rays of the angle; angle measures are limited to whole numbers**

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

ray
vertex of an angle

Role in Concept Development

Supports

4.7(C) determine the approximate measures of angles in degrees to the nearest whole number using a protractor

Connection/ Relevance

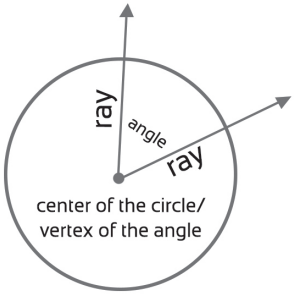
This supporting standard develops the understanding that an angle is a part of a circle that has been "cut out." The vertex of the angle serves as a point of interest in using the protractor to measure angles accurately.

When to Teach

With 4.7(C)

Instructional Implications

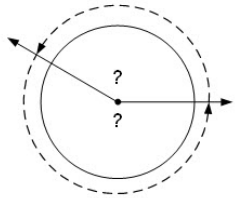
In conjunction with 4.6(A), students should identify the point in the middle of the circle that would be equal distance from that point to any point on the circle. This point would be identified as the center of the circle. The center of the circle serves as the vertex of two rays and the common endpoint of two rays (vertex of an angle). The part of the circle that is "cut out" by the rays is identified as the angle. Students should be able to illustrate such understanding.



Learning from Mistakes

Students may make the following mistakes:

- Confusing which angle to measure according to the "cut out" if not specifically marked (e.g., inside or outside angles, especially when both are obtuse)



4.7 Geometry and measurement. The student applies mathematical process standards to solve problems involving angles less than or equal to 180 degrees. The student is expected to:

4.7(B) **(B) illustrate degrees as the units used to measure an angle, where 1/360 of any circle is 1 degree and an angle that "cuts" *n*/360 out of any circle whose center is at the angle's vertex has a measure of *n* degrees; angle measures are limited to whole numbers**

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

angle
angle vertex
degree

Role in Concept Development

Supports
4.7(C) determine the approximate measures of angles in degrees to the nearest whole number using a protractor

Connection/ Relevance
This supporting standard develops the understanding that a complete circle represents 360°. Each angle unit represents $\frac{1}{360}$ of a circle. Students apply this knowledge when using a protractor which represents a half of a circle measuring angles up to 180°.

When to Teach
With 4.7(C)

Instructional Implications
Just like length is measured in linear units or time is measured in seconds/minutes/hours, angles are measured in degrees. Instruction may use the example of how a 360-degree turn on a skateboard relates to 360° in a circle (e.g., a skateboarder turns a complete circle in air on the skateboard to perform a 360; the turn all the way around a circle measures 360°). The vertex of the angle is the center of the circle (the point on which the skateboarder begins his rotation). Relating how far a rider turns his skateboard may support students' understanding.

Students must illustrate how each degree represents $\frac{1}{360}$ of a circle; therefore, a 180 on a skateboard would actually represent $\frac{180}{360}$ or a $\frac{1}{2}$ a turn of a circle or 180°. Should a skateboarder only be able to turn his/her board 90°, he/she completed a $\frac{90}{360}$ turn or $\frac{1}{4}$ of a complete circle.

Learning from Mistakes
Students may make the following mistakes:

- Not relating the (whole number) degrees in a circle to the fractional part of the circle cut by an angle
- Not understanding that certain circle cuts can be equivalent fractions (e.g., $\frac{3}{4}$ of a circle = $\frac{270}{360}$ or 270°)