TEKS Cluster: Exponential Functions

A.9 **Exponential functions and equations.** The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data.

Connected Knowledge and Skills A.12

Writing Exponential Functions

Readiness Standards

A.9(C) write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay

Supporting Standards

A.9(E) write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems

A.12(C) identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes

A.12(D) write a formula for the n^{th} term of arithmetic and geometric sequences, given the value of several of their terms

Describing Exponential Functions

Readiness Standards

A.9(D) graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems

Supporting Standards

A.9(A) determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities

A.9(B) interpret the meaning of the values of a and b in exponential functions of the form $f(x) = ab^x$ in real-world problems

A.12(A) decide whether relations represented verbally, tabularly, graphically, and symbolically define a function

TEKS Scaffold

TEKS	Student Expectation
2A.5(B)	formulate exponential and logarithmic equations that model real-world situations, including exponential relationships written in recursive notation (S)
A.9(C)	**A.9 Exponential functions and equations.** The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to: **(C) write exponential functions in the form *f(x)* = *ab*^{x} (where *b* is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay**

Stimulus

Word Problem*	Verbal Description	Chart/Table*	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor* (1-2 pts)	Multiple Choice* (1 pt)	

Content Builder (see Appendix for Tree Diagram)

- Write exponential functions in the form *f(x)* = *ab*^{x} (where *b* is a rational number)
- Describe problems arising from mathematical situations, including growth and decay
- Describe problems arising from real-world situations, including growth and decay

Instructional Implications

Students are expected to determine exponential functions in the form *f(x)* = *ab*^{x} from a variety of mathematical and real-world situations. In conjunction with A.12(D), instruction may begin with numbers in a geometric sequence, with an initial term and a common ratio (e.g., in the sequence 3, 6, 12, 24, 48..., the initial term is 3, and the common ratio is 2.) These attributes of geometric sequences can be compared to the values of *a* and *b* in exponential functions of the form *f(x)* = *ab*^{x}.

To clarify the meanings of the *a* and *b* values, instruction should include using technology to generate tables of values for various exponential functions. From this exploration, students should recognize that *a* is the value of *y* when *x* = 0 (the *y*-intercept), and *b* is the common ratio between consecutive *y*-values (when *x* increases by 1). For example:

Function: *f(x)* = 3(2)^x

x	0	1	2	3
f(x)	3	6	12	48

Values start at *a* = 3, then double (*b* = 2).

Function: *f(x)* = 10(0.5)^x

x	0	1	2	3
f(x)	10	5	2.5	1.25

Values start at *a* = 10, then decrease by a factor of *b* = 0.5.

Students should recognize the difference between exponential growth (the function values increase when *b* > 1) and exponential decay (the function values decrease when 0 < *b* < 1). From this process, instruction should lead students to write exponential functions from tables of values, as in the example below.

x	0	1	2	3
f(x)	40	60	90	1.25

f(0) = 40, so *a* = 40. 60/40 = 1.5, and 90/60 = 1.5, so *b* = 1.5. The function is *f(x)* = 40(1.5)^x.

(continued)

Instructional Implications (continued)

Instruction should also include real-world examples where students interpret situations or make tables to write exponential functions. For example:

Type	Exponential decay	Exponential growth
Situation	A piece of copy paper measures 8.5 in by 11 in. Imagine repeatedly folding the paper in half. With each fold, what happens to the area of each section of the paper?	Sam takes out a college loan for $10,000. He won't have to make any payments for 6 years, but during this time, interest compounds at an annual rate of 8%.
Table	x = number of folds $f(x)$ = area of each section (in^2) <table><tr><td>x</td><td>$f(x)$</td></tr><tr><td>0</td><td>8.5 x 11 = 93.5</td></tr><tr><td>1</td><td>46.75</td></tr><tr><td>2</td><td>23.375</td></tr><tr><td>3</td><td>11.6875</td></tr></table>	x = number of years $f(x)$ = amount owed ($) <table><tr><td>$x$</td><td>$f(x)$</td></tr><tr><td>0</td><td>10,000.00</td></tr><tr><td>1</td><td>10,800.00</td></tr><tr><td>2</td><td>11,664.00</td></tr><tr><td>3</td><td>12,597.12</td></tr></table>
Values	a = 93.5, b = 0.5	a = 10,000, b = 1.08
Function	$f(x) = 93.5(0.5)^x$	$f(x) = 10{,}000(1.08)^x$

Learning from Mistakes

Students may make the following mistakes:

- Confusing values of a and b in writing exponential functions*
- Thinking that exponential decay requires the value of b to be negative (instead of between 0 and 1)

Academic Vocabulary

decay
exponential function*
growth*

Interesting Items

A.9(C) 2024 #43
A.9(C) 2023 #14
A.9(C) 2017 #35
A.9(C) 2016 #28

 * Used on STAAR

A.9(E)	**A.9 Exponential functions and equations.** The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to: **(E) write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems**

Stimulus

Word Problem*	Verbal Description	Chart/Table*	Graph*
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

exponential function*
regression

Interesting Items

A.9(E) 2016 #17

Role in Concept Development

Supports

- A.9(C) write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay
- 2A.8(C) predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models

Connection/ Relevance

Using technology to generate models from data helps place exponential functions in real-world contexts. These skills are important for student success in later courses.

When to Teach

- After A.9(C)
- Before/Prerequisite to 2A.8(C)

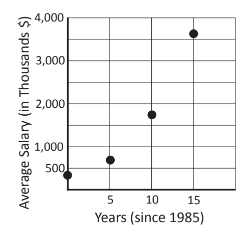
Instructional Implications

Instruction should include analyzing bivariate data collected from real-world situations and generating scatterplots. To model this data, students are expected to write an exponential equation using technology.

Students enter the data into a calculator or other device. Students analyze the shape of the scatterplot to verify that an exponential model is appropriate (as opposed to a linear or quadratic function). For example:

The table shows average salaries of NBA players by year.

Year	x	Avg. Salary (thousands $)
1985	0	330
1990	5	717
1995	10	1,800
2000	15	3,600



To generate an equation to match the data, students can use their devices to conduct an exponential regression. For example, the data in the situation above would have the following regression output:	Exponential Regression a•b^x a = 330.7 b = 1.17561

Using technology, the equation of best fit is given by $y = 330.7(1.17561)^x$.

(continued)

 * Used on STAAR

Role in Concept Development (continued)

Instructional Implications

Students are also expected to use this equation to make predictions. For example, in the previous situation, *x* represents time in years since 1985, and *y* is the average salary, in thousands of dollars, of an NBA athlete. If asked to estimate the average salary in 2005, students could evaluate the regression equation at $x = 20$, since 2005 is 20 years after 1985.

$y = 330.7(1.17561)^x$ When $x = 20$, $y = 330.7(1.17561)^{20} = 8408.13$ According to this model, the average salary would be $8,408,130. Note: Actually, while average player salaries in the NBA grew exponentially from 1980 to 2000, they have not increased much since then. The current average is around $4,000,000.

Learning from Mistakes

Students may make the following mistakes:

- Incorrectly recording or entering data
- Misinterpreting the values of *a* and *b* in $f(x) = ab^x$

A.12(C)

A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

(C) identify terms of arithmetic and **geometric sequences when the sequences are given in function form using recursive processes**

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

common ratio
geometric sequence

Interesting Items

A.12(C) 2017 #22

Role in Concept Development

Supports

A.9(C) write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay

Connection/ Relevance

Geometric sequences create simple patterns that relate directly to exponential functions. The relationship between the number of the term (n) and the term itself (a_n) mirrors the relationship between x and y (domain and range). The common ratio (r) of the sequence acts just like the base (b) of the exponential function.

When to Teach

- With A.9(C), if teaching geometric sequences (only) in conjunction with exponential functions
- After A.9(C), if teaching both arithmetic and geometric sequences together, after teaching linear and exponential functions separately

Instructional Implications

NOTE: The instructional considerations below provide guidance for using only geometric sequences (not arithmetic) when clustered with exponential functions. For more information about arithmetic sequences, see A.12(C) in the Linear Functions TEKS Cluster.

Instruction should include generating the terms of geometric sequences from a given function. In geometric sequences, terms increase/decrease by the same factor each time. This multiplier is called the common ratio. For example, in the geometric sequence 40, 20, 10, 5, 2.5,..., the first term (a_1) is 40, the second term (a_2) is 20, etc., and the common ratio is 0.5.

Students should be able to use a function to generate terms in a sequence. For example, consider the function $a(n) = 3(2)^{n-1}$. If n represents the number of the term in a sequence, then evaluating at n = 1, 2, 3... will generate the sequence $a_1 = 3$, $a_2 = 6$, $a_3 = 12$, etc.

Recognizing a geometric sequence with a common ratio of 2, students should be able to extend the sequence recursively by multiplying by 2 each time (3, 6, 12, 24, 48, 96, 192,...).

Learning from Mistakes

Students may make the following mistakes:

- Dividing terms in the wrong order when computing the common ratio
- Confusing the number of the term (n) with the term itself (a_n)*

* Used on STAAR

A.12(D) **A.12 Number and algebraic methods.** The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

(D) write a formula for the n^{th} term of arithmetic and **geometric sequences, given the value of several of their terms**

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

common ratio
geometric sequence

Interesting Items

Data included in Linear Functions

Role in Concept Development

Supports

A.9(C) write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay

Connection/ Relevance

Geometric sequences create simple patterns that relate directly to exponential functions. The relationship between the number of the term (n) and the term itself (a_n) mirrors the relationship between x and y (domain and range). The common ratio (r) of the sequence acts just like the base (b) of the exponential function. Students should be able to compare the first term and common ratio in geometric sequences to the values of a and b in exponential functions of the form $f(x) = ab^x$.

When to Teach

- With A.9(C), if teaching geometric sequences (only) in conjunction with exponential functions
- After A.9(C), if teaching both arithmetic and geometric sequences together, after teaching linear and exponential functions separately

Instructional Implications

NOTE: The instructional considerations below provide guidance for using only equations. For more information about arithmetic sequences, see A.12(D) in the Linear Functions TEKS Cluster.

Instruction should include developing the formulas to find any term of a geometric sequence. In geometric sequences, terms increase/decrease by the same factor each time. This multiplier is called the common ratio. For example, in the geometric sequence 40, 20, 10, 5, 2.5,..., the common ratio (r) can be found by dividing any pair of consecutive terms. Here, the common ratio is 0.5, since a_2/a_1 = 20/40 = 0.5, or a_3/a_2 = 10/20 = 0.5.

After defining each of these measures, students should be asked how to find an unknown term in the sequence, such as the 50th term. A sample response would be, "Start with the first term (40) and multiply by the common ratio (0.5) 49 times." From this, students can develop the formula to find any term in a geometric sequence: $a(n) = a_1(r)^{n-1}$. Here the formula for the sequence 40, 20, 10, 5, 2.5... would be $a(n) = 40(0.5)^{n-1}$.

Learning from Mistakes

Students may make the following mistakes:

- Confusing the number of the term (n) with the term itself (a^n)
- Using the recursive formula instead of the explicit formula (e.g., given {2, 6, 18, 54...} mistakenly writing the equation as $a_n = 3n$)

* Used on STAAR

TEKS Scaffold

TEKS	Student Expectation
2A.2(A)	graph the functions $f(x)=\sqrt{x}$, $f(x)=1/x$, $f(x)=x^3$, $f(x)=\sqrt[3]{x}$, $f(x)=bx$, $f(x)=\lvert x \rvert$, and $f(x)=\log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval (R)
A.9(D)	**A.9 Exponential functions and equations.** The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to: **(D) graph exponential functions that model growth and decay and identify key features, including *y*-intercept and asymptote, in mathematical and real-world problems**
A.3(C)	graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems (R)

Stimulus

Word Problem*	Verbal Description*	Chart/Table	Graph*
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Content Builder (see Appendix for Tree Diagram)

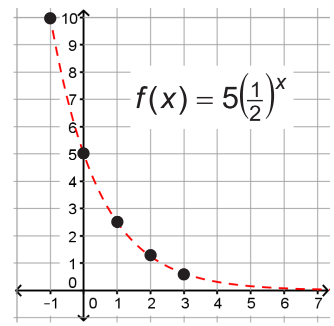
- Graph exponential functions that model growth and decay
- Identify key features, including *y*-intercept and asymptote, in mathematical problems
- Identify key features, including *y*-intercept and asymptote, in real-world problems

Instructional Implications

Students are expected to graph exponential functions and identify key features in a variety of problem situations. Instruction may include plotting points from a table of values generated from an exponential function. For example:

$f(x) = 5(\frac{1}{2})^x$

x	*f(x)*
-1	$5(\frac{1}{2})^{-1} = 5 \div (\frac{1}{2}) = 5(2) = 10$
0	$5(\frac{1}{2})^0 = 5(1) = 5$
1	$5(\frac{1}{2})^1 = 5(\frac{1}{2}) = 2.5$
2	$5(\frac{1}{2})^2 = 5(\frac{1}{2})(\frac{1}{2}) = 1.25$
3	$5(\frac{1}{2})^3 = 5(\frac{1}{2})(\frac{1}{2})(\frac{1}{2}) = 0.625$



In the example above, students should be able to identify the *y*-intercept as the point where $x = 0$ (in this example, (0, 5)). Students should also notice that as *x* increases, the function values get closer and closer to zero but will never reach it. This pattern can be used to describe the function's asymptote, which is $y = 0$ (the *x*-axis).

Instruction should also include the use of graphing technology as a way to predict graphical behavior from the constants used in the exponential function.

$f(x) = ab^x$	*y*-intercept	Increasing (growth) or Decreasing (decay)?	asymptote
$f(x) = 3(1.5)^x$	(0, 3)	Increasing (growth)	$y = 0$ (*x*-axis)
$f(x) = 0.5(2)^x$	(0, 0.5)	Increasing (growth)	$y = 0$ (*x*-axis)
$f(x) = 2(0.8)^x$	(0, 2)	Decreasing (decay)	$y = 0$ (*x*-axis)

When *a* and *b* are positive rational numbers, functions of the form $f(x) = ab^x$ will:

- Have *y*-intercepts at (0, *a*)
- Show exponential growth if $b > 1$
- Show exponential decay when $0 < b < 1$
- Have a horizontal asymptote at $y = 0$

These features will assist in the graphing of exponential functions.

(continued)

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop* (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Instructional Implications (continued)

When graphing exponential functions that model real-world situations, students may have to adjust the scales on the *x* and *y* axes or calculator's window to values that make sense in the problem.

For example, a tablet computer is purchased for $500 but only retains 75% of its value every year; the function $f(x) = 500\,(0.75)^x$ represents the situation. To graph this function, students should recognize that *x* represents time in years after a tablet is purchased, so the graph may only need to cover values from $x = 0$ to $x = 8$. Students should recognize that *y* represents the tablet's decreasing value over time, so the graph may only need to cover values up to $y = 500$.

Learning from Mistakes

Students may make the following mistakes:
- After plotting a few points of an exponential decay function, thinking that values will eventually become negative instead of approaching zero
- Confusing a constant ratio (such as *b* in $f(x) = ab^x$) with a constant rate of change (such as *m* in $y = mx + b$)
- Confusing the independent and dependent variables in a real-world problem situation
- Identifying the exponential values raised to the zero power as a value of one [e.g., $3(4)^0 = 3(4)(1) = 12$] instead of a value raised to the zero power as having a value of one [e.g., $3(4)^0 = 3(1) = 3$]*
- Confusing the asymptote and *y*-intercept*
- Misidentifying the asymptote as the vertical instead of horizontal*
- Identifying key attributes in an exponential function when no graph is given*

Academic Vocabulary

asymptote*
decay
exponential function*
growth
x-intercept*
y-intercept*

Interesting Items

A.9(D) 2018 #40
A.9(D) 2017 #49
A.9(D) 2016 #50

A.9(A) **A.9 Exponential functions and equations.** The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(A) determine the domain and range of exponential functions of the form *f(x)* = ab^x and represent the domain and range using inequalities

Stimulus

Word Problem	Verbal Description*	Chart/Table	Graph*
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect* (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

asptote
domain*
exponential function*
range*

Interesting Items

A.9(A) 2023 #18
A.9(A) 2017 #21
A.9(A) 2016 #21

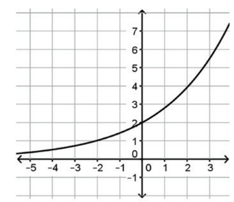
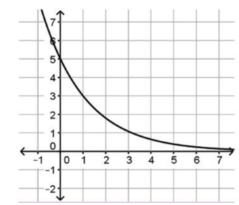
Role in Concept Development

Supports
A.9(D) graph exponential functions that model growth and decay and identify key features, including *y*-intercept and asymptote, in mathematical and real-world problems

Connection/ Relevance
Students must identify the domain and range of exponential functions as the range of an exponential function relates graphically to the function's horizontal asymptote.

When to Teach
With A.9(D)

Instructional Implications
Instruction should include determining the domain and range of exponential functions. In conjunction with A.2(A) and A.6(A), instruction should begin by defining domain as the set of *x*-values used by a function and range as the set of *y*-values. Students may determine domain and range from graphs.

Graph		
Domain	All real numbers	All real numbers
Range	y > 0	y > 0

For continuous exponential functions of the form *f(x)* = ab^x (which requires *b* > 0), the mathematical domain will always be the set of all real numbers. As long as *a* > 0, the range for such exponential functions will be *y* > 0 because of the asymptotic behavior along the *x*-axis.

In addition to identifying the domain and range of a graph, students are also expected to interpret limitations on independent and dependent variables in real-world problems using exponential functions.

(continued)

Role in Concept Development (continued)

Instructional Implications

For example, certain situations may require the use of only positive numbers, which restricts the values in the domain and/or range.

Situation	A scientist grows a culture of bacteria in a lab. Initially, there are 50 bacteria, but the population grows by a factor of 1.5 each week.
Function	$P(t) = 50(1.5)^t$
Domain	$t \geq 0$ Values of time (in weeks) must be positive or zero.
Range	$P(t) \geq 50$ The population of bacteria starts at 50 and increases each week.

Situation	A new phone is initially purchased for $200. Over the next 12 months, the phone only retains 90% of its value per month.
Function	$V = 200(0.9)^x$
Domain	$0 \leq x \leq 12$ The function only estimates the phone's value between 0 and 12 months.
Range	$56.49 \leq V \leq 200$ The phone's value is initially $200 but drops to $56.49 after 12 months.

Learning from Mistakes

Students may make the following mistakes:

- Switching or confusing the *x* and *y* variable
- Choosing the incorrect inequality symbol to use
- Misinterpreting the domain and/or range for continuous data represented on a graph (e.g., range: $0 < x < 500$ as that is what is visually displayed on the graph; however, the true range is all real numbers > than 500)*

A.9(B) Supporting

Subcluster: Describing Exponential Functions

A.9(B) **A.9 Exponential functions and equations.** The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data.
The student is expected to:

(B) interpret the meaning of the values of *a* and *b* in exponential functions of the form $f(x) = ab^x$ in real-world problems

Stimulus

Word Problem*	Verbal Description*	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

coefficient
exponential function*

Interesting Items

A.9(B) 2023 #27
A.9(B) 2018 #46

Role in Concept Development

Supports

- A.9(C) write exponential functions in the form $f(x) = ab^x$ (where *b* is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay
- A.9(D) graph exponential functions that model growth and decay and identify key features, including *y*-intercept and asymptote, in mathematical and real-world problems

Connection/ Relevance

Students will need to associate the meaning of the values of *a* and *b* of exponential functions that are in the form of $f(x) = ab^x$ in order to effectively write and graph the functions.

When to Teach

With A.9(C) and A.9(D)

Instructional Implications

Instruction should include analysis of how values of *a* and *b* affect exponential functions in the form $f(x) = ab^x$ in a variety of mathematical and real-world situations. In conjunction with A.12(D), instruction may begin with numbers in a geometric sequence, with an initial term and a common ratio (e.g., in the sequence 3, 6, 12, 24, 48..., the initial term is 3, and the common ratio is 2.) These attributes of geometric sequences can be compared to the values of *a* and *b* in exponential functions of the form $f(x) = ab^x$.

To clarify the meanings of the *a* and *b* values, instruction should include using technology to generate tables of values for various exponential functions. From this exploration, students should recognize that *a* is the value of *y* when $x = 0$ (the *y*-intercept), and *b* is the common ratio (when *x* increases by 1). For example:

Function: $f(x) = 3(2)^x$

x	0	1	2	3
$f(x)$	3	6	12	48

Values start at $a = 3$, then double ($b = 2$).

Function: $f(x) = 10(0.5)^x$

x	0	1	2	3
$f(x)$	10	5	2.5	1.25

Values start at $a = 10$, then decrease by a factor of $b = 0.5$.

Students should recognize the difference between exponential growth (the function values increase when $b > 1$) and exponential decay (the function values decrease when $0 < b < 1$).

Students should also be able to interpret the values of *a* and *b* in real-world contexts. For example, suppose the population (*p*, in thousands of people) of a town can be modeled using $p = 50(1.04)^t$, where *t* is time in years. From the value of $a = 50$, students should be able to identify the town's initial population (or, when $t = 0$, the population was 50,000). From the value of $b = 1.04$, students should interpret that the population is increasing (since $b > 1$) at a rate of 4% growth each year.

Learning from Mistakes

Students may make the following mistakes:
- Switching or confusing the roles of *a* and *b*
- Incorrectly identifying whether the function shows growth or decay

* Used on STAAR

A.12(A) **A.12 Number and algebraic methods.** The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

(A) decide whether relations represented verbally, tabularly, graphically, and symbolically define a function

Stimulus

Word Problem	Verbal Description	Chart/Table*	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

domain
range

Interesting Items

A.12(A) 2017 #38

Role in Concept Development

Supports
A.9(D) graph exponential functions that model growth and decay and identify key features, including *y*-intercept and asymptote, in mathematical and real-world problems

Connection/ Relevance
Much of Algebra I is based on the identification of functions and functional relationships. Students must be able to identify the relationship between the domain and range of exponential functions in order to appropriately write, solve, analyze and evaluate functions and related graphs.

When to Teach
Before/Prerequisite to A.9(D)

Instructional Implications
Instruction should include defining a function as a relation where each element in the domain is paired with exactly one element in the range and applying this definition in multiple representations.

Students should be able to identify functions from a table of *x*- and *y*-values. If a relation is a function, then no *x*-value will be repeated in the table.

This is NOT a function:

x	*y*
5	1
6	2
7	3
6	4

(An *x*-value repeats; for $x = 6$, there are two possible *y*-values.)

This IS a function:

x	*y*
1	5
2	6
3	7
4	6

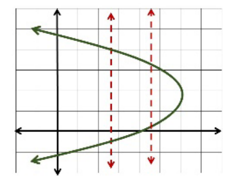
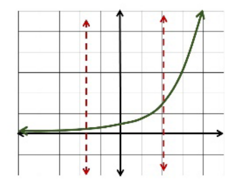
(No *x*-value repeats; there is only one *y*-value for each *x*.)

(continued)

Role in Concept Development (continued)

Instructional Implications

Students should also be able to recognize a function from the graph. On the coordinate plane, functions will pass the “vertical line test,” which means that any vertical line will intersect the graph of a function at no more than one point.

If the graph of a relation has two points that are “stacked” atop one another, the relation is not a function (and will fail the vertical line test).	This is NOT a function.  (A vertical line crosses the graph at more than one point.)	Graphs showing exponential growth or decay ARE functions.  (Any vertical line will only cross the graph once.)

Symbolically, relations are functions if they can be written as a single equation in “y =” form. In the cluster for exponential functions, students work only with equations of the form $y = ab^x$. As the “y =” form indicates, all equations written in this manner are functions.

Students should also be able to identify whether a relation is a function from a verbal description, by applying the definition to the *x*- and *y*-variables represented in the situation. For example:

Description	In a high school, each student’s grade level (*x*) is paired with the student’s height (*y*) in inches.	In a high school, each student’s ID number (*x*) is paired with the student’s grade level (*y*).
Is it a function?	NO	YES
Reason	Many students will be in the same grade but have different heights. So, the same *x*-value will be paired with many different *y*-values.	Each student has a unique ID number that will not be repeated, and a student will not be classified in more than one grade level.

Learning from Mistakes

Students may make the following mistakes:

- Switching *x* and *y* values (or switching domain and range)
- Confusing the rule about repeated *y*-values (which can be a function) with repeated *x*-values (which is not a function)