TEKS Cluster: Quadratic Functions

A.6 **Quadratic functions and equations.** The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations.

A.7 **Quadratic functions and equations.** The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations.

A.8 **Quadratic functions and equations.** The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data.

Connected Knowledge and Skills A.12

Writing and Solving Quadratic Equations

Readiness Standards

A.8(A) solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula

Supporting Standards

A.6(B) write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form ($f(x) = a(x - h)^2 + k$), and rewrite the equation from vertex form to standard form ($f(x) = ax^2 + bx + c$)

A.6(C) write quadratic functions when given real solutions and graphs of their related equations

A.7(B) describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions

A.8(B) write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems

Describing Quadratic Functions

Readiness Standards

A.6(A) determine the domain and range of quadratic functions and represent the domain and range using inequalities

A.7(A) graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry

A.7(C) determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d

Supporting Standards

A.12(A) decide whether relations represented verbally, tabularly, graphically, and symbolically define a function

A.12(B) evaluate functions, expressed in function notation, given one or more elements in their domains

TEKS Scaffold

TEKS	Student Expectation
2A.4(F)	solve quadratic and square root equations (R)
A.8(A)	**A.8 Quadratic functions and equations.** The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to: **(A) solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula**
A.5(A)	solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides (R)

Stimulus

Word Problem*	Verbal Description	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Content Builder (see Appendix for Tree Diagram)

- Solve quadratic functions by factoring
- Solve quadratic functions by taking square roots
- Solve quadratic functions by completing the square
- Solve quadratic functions by applying the quadratic formula

Instructional Implications

Students are expected to solve quadratic equations using a variety of methods. Instruction may begin with solving simple equations involving an x^2 term by taking square roots (e.g., if $x^2 + 9 = 25$; $x^2 = 16$; so $x = \sqrt{16} = 4$).

While students may be familiar with this type of equation from Grade 8 (because of the Pythagorean theorem), instruction in Algebra I should now emphasize that these equations have two solutions (e.g., if $x^2 = 16$; $x = \pm\sqrt{16}$; x can equal 4 or -4).

Instruction can then proceed to multi-step equations that can be solved using square roots, such as the sample provided at right.

$$2(x-3)^2 - 5 = 27$$
$$2(x-3)^2 = 32$$
$$(x-3)^2 = 16$$
$$x-3 = \pm\sqrt{16} = \pm 4$$
$$x = 3 \pm 4 = 7 \text{ or } -1$$

Next, instruction should include quadratic equations with both x^2 and x terms, such as $x^2 + 6x = 27$. Since equations in this form cannot be directly solved with square roots, the processes of factoring [see A.10(E)] and completing the square can be used to find solutions.

Factoring	Completing the Square
$x^2 + 6x = 27$ $x^2 + 6x - 27 = 0$ $(x + 9)(x - 3) = 0$ $x + 9 = 0$, or $x - 3 = 0$ $x = -9$, or $x = 3$	$x^2 + 6x = 27$ Think: $\frac{1}{2}(6) = 3$, $(3)^2 = 9$. Add 9 to both sides. $x^2 + 6x + 9 = 27 + 9$ $(x + 3)^2 = 36$ $x + 3 = \pm\sqrt{36} = \pm 6$ $x = -3 \pm 6$ $x = -3 + 6 = 3$, or $x = -3 - 6 = -9$

Lastly, instruction should include the quadratic formula as yet another way to solve these equations. This formula uses the coefficients of a quadratic equation in the form $ax^2 + bx + c = 0$ to determine the solutions.

Quadratic Formula	Sample
If $ax^2 + bx + c = 0$, and $a \neq 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Solve: $2x^2 + 6x = 7$. Set equal to zero: $2x^2 + 6x - 7 = 0$ So, $a = 2$, $b = 6$, $c = -7$. $x = \frac{-6 \pm \sqrt{(-6)^2 - 4(2)(-7)}}{2(2)}$ $x = \frac{-6 \pm \sqrt{92}}{4} = \frac{-6 \pm 2\sqrt{23}}{4}$ $x = \frac{-3 \pm \sqrt{23}}{2}$

(continued)

 * Used on STAAR

Learning from Mistakes

Students may make the following mistakes:

- When factoring, confusing whether pairs of numbers have a common product or common sum
- Not knowing to set equations equal to 0 before solving
- Making sign errors when determining solutions from factors*
- When evaluating the quadratic formula, making arithmetic mistakes involving integers, squares, square roots, fractions, and/or the order of operations*
- When using the quadratic formula, incorrectly simplifying (or forgetting to simplify) the radical expression
- Having difficulty remembering how to complete the square due to lack of understanding of the process of factoring perfect square trinomials

Academic Vocabulary

complete the square
factor
no real solution*
quadratic equation*
quadratic formula
take square roots

Interesting Items

A.8(A) 2024 #31
A.8(A) 2023 #41
A.8(A) 2021 #37
A.8(A) 2018 #41
A.8(A) 2016 #22

A.6 Quadratic functions and equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:

A.6(B) **(B) write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form ($f(x) = a(x - h)^2 + k$), and rewrite the equation from vertex form to standard form ($f(x) = ax^2 + bx + c$).**

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

minimum point*
quadratic function
standard form (of a quadratic function)
vertex
vertex form (of a quadratic function)

Interesting Items

A.6(B) 2023 #25
A.6(B) 2018 #26

Role in Concept Development

Supports

- A.7(A) graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum value, vertex, and the equation of the axis of symmetry
- A.7(C) determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d

Connection/ Relevance

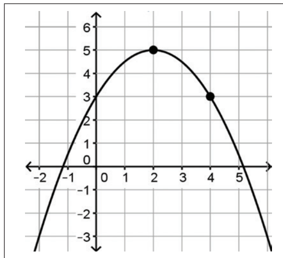
The vertex form may be taught in conjunction with graphing transformations of the parent function $f(x) = x^2$, as values of h and k relate to translating this graph to the left/right and up/down.

When to Teach

With A.7(A) and A.7(C)

Instructional Implications

Instruction should include writing equations for quadratic functions in vertex form, $f(x) = a(x - h)^2 + k$, where the coordinates for the vertex are (h, k). While the vertex will determine values for h and k, another point on the graph of the quadratic function is necessary to determine the value of a, as demonstrated in the example below.

	A quadratic function has its vertex as (2, 5) and also passes through (4, 3). Write its equation in vertex form.	$f(x) = a(x - h)^2 + k$ $(h, k) = (2, 5)$ $f(x) = a(x - 2)^2 + 5$ $(4, 3)$ means $f(4) = 3$. $3 = a(4 - 2)^2 + 5$ $3 = 4a + 5$ $a = -0.5$

The equation for this function in vertex form is $f(x) = -0.5(x - 2)^2 + 5$.

In addition to writing the equation in vertex form, students are expected to rewrite the equation in standard form ($f(x) = ax^2 + bx + c$). Students must square the binomial, distribute the value of a, and combine like terms, as shown in the example at right.

$f(x) = -0.5(x - 2)^2 + 5$
$f(x) = -0.5(x - 2)(x - 2) + 5$
$f(x) = -0.5(x^2 - 4x + 4) + 5$
$f(x) = -0.5x^2 + 2x - 2 + 5$
$f(x) = -0.5x^2 + 2x + 3$

Learning from Mistakes

Students may make the following mistakes:
- Confusing the signs for h and k (since h follows a minus sign and k follows a plus sign)
- In rewriting equations from vertex form to standard form, making errors in squaring the binomial*
- Having difficulty identifying equivalent quadratic expressions [$(x+4)^2 = x^2 + 16$ instead of $x^2 + 8x + 16$]*

 * Used on STAAR

A.6(C)	**A.6 Quadratic functions and equations.** The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to: **(C) write quadratic functions when given real solutions and graphs of their related equations**

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph*
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

factor
quadratic function
solution
x-intercept
zeros

Interesting Items

A.6(C) 2018 #7
A.6(C) 2017 #10

Role in Concept Development

Supports

- A.7(A) graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum value, vertex, and the equation of the axis of symmetry
- A.8(A) solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula

Connection/ Relevance

Students will be solving quadratic equations using factoring. When solving quadratic equations in this way, students should recognize that if an equation factors into the form $(x - m)(x - n) = 0$, then the values m and n are solutions to the equation. The converse of this statement is also true, which assists in writing functions and can be verified graphically.

When to Teach

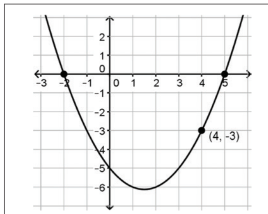
After A.7(A) / With A.8(A)

Instructional Implications

Instruction should include writing quadratic functions from the solutions to their related equations. In general, if the numbers m and n are solutions to a quadratic function, then $(x - m)$ and $(x - n)$ are factors. The solutions to the related equation can provide information to write the quadratic function; however, students will still need another point on the function to determine the leading coefficient, a. In general, this relationship can be described as:

> If a quadratic function has solutions (or zeros, or *x*-intercepts) at $x = m$ and $x = n$, then its equation can be written as $f(x) = a(x - m)(x - n)$.

For example:



Write the equation for the quadratic function that passes through (4, -3) and has zeros at $x = -2$ and $x = 5$.

$f(x) = a(x - m)(x - n)$
$f(x) = a(x + 2)(x - 5)$
(4, -3 means $f(4) = -3$.
$-3 = a(4 + 2)(4 - 5)$
$-3 = a(6)(-1)$
$0.5 = a$
So, $f(x) = 0.5(x + 2)(x - 5)$)

From this factored form, students may be required to rewrite the equation in standard form ($f(x) = ax^2 + bx + c$). Students must multiply the binomials, distribute the value of a, and combine like terms, as shown in the example below.

$f(x) = 0.5(x + 2)(x - 5)$
$f(x) = 0.5(x^2 - 5x + 2x - 10)$
$f(x) = 0.5(x^2 - 3x - 10)$
$f(x) = 0.5x^2 - 1.5x - 5$

Learning from Mistakes

Students may make the following mistakes:
- Making sign errors in writing the factors from the solutions
- Forgetting the need to determine the value of a as the lead coefficient of the quadratic function

* Used on STAAR

A.7(B)

A.7 Quadratic functions and equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:

(B) describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions

Stimulus

Word Problem	Verbal Description*	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop* (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

factor
quadratic function
x-intercept
zeros*

Interesting Items

A.7(B) 2024 #34
A.7(B) 2023 #8

Role in Concept Development

Supports

A.8(A) solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula

Connection/ Relevance

When solving quadratic equations by factoring, students should recognize that if an equation factors into the form $(x - m)(x - n) = 0$, then the values m and n are solutions to the equation.

When to Teach

With – context

Instructional Implications

Instruction should include descriptions of the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions. This skill centers on the zero product rule (If $A \bullet B = 0$, then either $A = 0$ or $B = 0$). In solving quadratic equations with this rule, students should recognize that if an equation factors into the form $(x - m)(x - n) = 0$, the values m and n are solutions (or zeros) to the equation. From this, the converse statement is also true: If the numbers m and n are solutions to a quadratic function, then $(x - m)$ and $(x - n)$ are factors.

In addition, students should recognize that quadratic equations are also affected by the leading coefficient, a. In general, this relationship can be described as:

If a quadratic equation has solutions (or zeros) at $x = m$ and $x = n$, then the related expression can be factored as $a(x - m)(x - n)$.

Learning from Mistakes

Students may make the following mistakes:
- Making sign errors in writing the factors from the solutions
- Forgetting the need to determine the value of a as the lead coefficient of the quadratic function

A.8(B)

A.8 Quadratic functions and equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(B) write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems

Stimulus

Word Problem	Verbal Description	Chart/Table*	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

quadratic function
regression
x-intercept*
y-intercept*
vertex (of a quadratic function)*

Interesting Items

A.8(B) 2023 #13
A.8(B) 2021 #47
A.8(B) 2017 #39

Role in Concept Development

Supports

- A.7(A) graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum value, vertex, and the equation of the axis of symmetry
- A.7(C) determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d
- 2A.8(C) predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models

Connection/ Relevance

Using technology to generate models from data helps place graphs and equations for quadratic functions in real-world contexts.

When to Teach

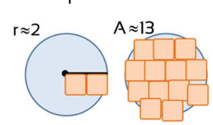
- After A.7(A) and A.7(C)
- Before/Prerequisite to 2A.8(C)

Instructional Implications

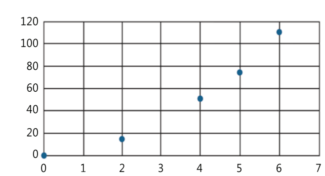
Instruction should include analyzing bivariate data collected from real-world situations. To model this data, students are expected to write a quadratic equation using technology.

To determine an equation to fit data, students can enter the data into a calculator or other device to generate a scatterplot. The shape of the graph should help students verify that a quadratic model is appropriate, as opposed to a linear or exponential function. For example, consider the situation below:

Using circles of various sizes, students estimate each radius and area using square cereal pieces.



Radius	Area
0	0
2	13
4	50
5	74
6	110



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Role in Concept Development (continued)

Instructional Implications

To generate an equation to match the data, students may begin by graphing the quadratic parent function ($f(x) = x^2$) and transforming it using various constants in the equation [see A.7(C)]. For more precision, students can use their devices to conduct a quadratic regression. For example, the data in the situation above would have the following regression output:

Quadratic Regression $a \bullet x^2 + b \bullet x + c$
$a = 2.96962$
$b = 0.326759$
$c = 0.181237$

Using technology, the equation of best fit is given by $y = 2.96962x^2 + 0.326759x + 0.181237$.

Students are also expected to use this equation to make predictions. For example, in the situation above, x represents the radius measurements, and y gives the area (in cereal pieces). If asked to estimate how many cereal squares it would take to cover a circle with a radius of 10 units, students could evaluate the regression equation at $x = 10$.

$$y = 2.96962x^2 + 0.326759x + 0.181237$$
When $x = 10$,
$$y = 2.96962(10)^2 + 0.326759(10) + 0.181237$$
$$y = 300.411$$
It would take about 300 squares to cover the circle.

Learning from Mistakes

Students may make the following mistakes:

- Making errors in data entry
- Confusing the x and y variables, or misinterpreting their meaning in the context

TEKS Scaffold

TEKS	Student Expectation
2A.7(I)	write the domain and range of a function in interval notation, inequalities, and set notation (S)
A.6(A)	**A.6 Quadratic functions and equations.** The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to: **(A) determine the domain and range of quadratic functions and represent the domain and range using inequalities**
8.5(G)	identify functions using sets of ordered pairs, tables, mappings, and graphs (R)

Stimulus

Word Problem	Verbal Description*	Chart/Table*	Graph*
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

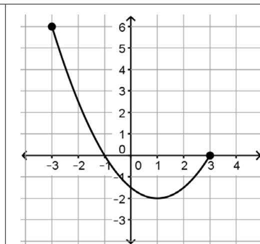
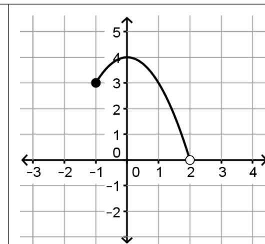
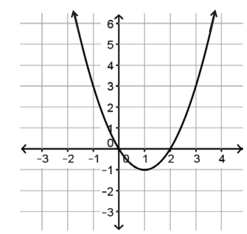
Content Builder (see Appendix for Tree Diagram)

- Determine the domain of a quadratic function in mathematical problems
- Determine the range of a quadratic function in mathematical problems
- Represent the domain and range using inequalities

Instructional Implications

Students are expected to determine the domain and range of quadratic functions. In conjunction with A.2(A), instruction should begin by defining domain as the set of *x*-values used by a function and range as the set of *y*-values.

Students determine domain and range from graphs. For continuous quadratic functions, inequalities should be used to represent the domain and range.

Graph			
Domain	$-3 \le x \le 3$	$-1 \le x < 2$	All real numbers
Range	$-2 \le y \le 6$	$0 < y \le 4$	$y \ge -1$

In most cases, describing the range implies identifying the *y*-coordinate of the graph's vertex, since this number relates to the maximum or minimum value of the function.

(continued)

Instructional Implications (continued)

In addition to identifying the domain and range of a graph, students are expected to interpret limitations on independent and dependent variables in real-world problems. For example, certain situations may require the use of only positive numbers, which restricts the values in the domain and/or range.

Situation	An object is dropped from a height of 100 ft. Its height (h, in feet) after t seconds can be found using $h(t) = 100 - 16t^2$.	Charging too much for a product can actually hurt profits. A marketing company estimates that charging x dollars for a certain product will generate R dollars in revenue according to the function $R = -12.5x^2 + 150x$.
Vertex	(0, 100)	(6, 450)
Zero(s)	(2.5, 0)	(0, 0) and (12, 0)
Domain	$0 \le t \le 2.5$ The object falls for only 2.5 seconds.	$0 \le x \le 12$ The price of the product should be between $0 and $12.
Range	$0 \le h(t) \le 100$ The height of the object will be 100 ft or less (but not negative).	$0 \le R \le 450$ The product will generate between $0 and $450 in revenue.

Learning from Mistakes

Students may make the following mistakes:

- Confusing *x* and *y* values*
- Confusing which inequality symbol to use (< or >, ≤ or ≥, etc.)
- Having difficulty determining the domain or range for a given table of values*
- Having trouble interpreting the independent and dependent variables in a real-world situation
- Confusing domain and range of quadratic functions*
- Having difficulty representing a graph when given a limited domain/range*
- Having difficulty determining how a problem situation can limit the domain or the range
- Identifying the spread of the quadratic function represented on a graph as the domain instead of the *x*-intercepts*

Academic Vocabulary

domain*
quadratic function*
range*

Interesting Items

A.6(A) 2024 #9
A.6(A) 2024 #32
A.6(A) 2023 #11
A.6(A) 2022 #47
A.6(A) 2016 #12

TEKS Scaffold

TEKS	Student Expectation
2A.2(A)	graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = \lvert x \rvert$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval (R)
A.7(A)	**A.7 Quadratic functions and equations.** The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to: **(A) graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry**
A.3(C)	graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems (R)

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph*
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect* (2 pts)	Match Table Grid (2 pts)	Drag and Drop* (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Content Builder (see Appendix for Tree Diagram)

- Graph quadratic functions on the coordinate plane
- Identify key attributes of graphs of quadratic functions, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry

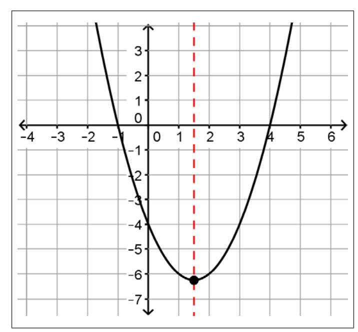
Instructional Implications

Students are expected to graph quadratic functions and identify the key attributes. In addition to using technology such as graphing calculators, students should also graph functions by hand on a coordinate grid, typically by plotting points generated from a table of values for the function. After defining each of the key attributes of quadratic functions, instruction should include identifying these features both graphically and algebraically. A summary is included below for quadratic functions of the form $f(x) = ax^2 + bx + c$.

Attribute	Graphical Definition	Algebraic Application
y-intercept	Point where the function crosses the *y*-axis or the point with an *x*-coordinate of 0.	Evaluate the function when $x = 0$. Since $f(0) = c$, the *y*-intercept is $(0, c)$.
x-intercept	Point(s) where the function crosses the *x*-axis or the point(s) with a *y*-coordinate of 0.	Set $f(x) = 0$ and solve. In general, the zeros can be found using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Zeros	Values of x that make $f(x) = 0$.	
Axis of symmetry	The vertical line through the “center” of the graph.	The formula for the axis of symmetry is: $x = -\frac{b}{2a}$
Vertex	The highest or lowest point on the graph of a quadratic function.	Evaluate the function when $x = -\frac{b}{2a}$. The coordinates for the vertex are $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
Maximum or minimum values	The *y*-coordinate of the vertex.	Evaluate the function when $x = -\frac{b}{2a}$, or find $f\left(-\frac{b}{2a}\right)$

See the examples below.

Function: $f(x) = x^2 - 3x - 4$
y-intercept: (0, -4)
x-intercepts: (-1, 0) and (4, 0)
zeros: $x = -1$, $x = 4$
Axis of symmetry: $x = 1.5$
Vertex: (1.5, -6.25)
Maximum value: none
Minimum value: $y = -6.25$



(continued)

Learning from Mistakes

Students may make the following mistakes:

- Confusing *x*-intercepts and the *y*-intercept, especially in terms of which of the coordinates is equal to zero*
- Not considering that a quadratic function can have one or two *x*-intercepts or no *x*-intercepts at all
- Having difficulty determining properties of symmetry
- Confusing the vertex with the axis of symmetry*

Academic Vocabulary

axis of symmetry*
maximum value*
minimum value*
parabola*
quadratic function*
vertex (of a quadratic function)*
x-intercept*
y-intercept*
zeros*

Interesting Items

A.7(A) 2023 #49
A.7(A) 2022 #1
A.7(A) 2019 #46

TEKS Scaffold

TEKS	Student Expectation
2A.4(C)	determine the effect on the graph of $f(x) = \sqrt{x}$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(bx)$, and $f(x - c)$ for specific positive and negative values of a, b, c, and d (R)
A.7(C)	**A.7 Quadratic functions and equations.** The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to: **(C) determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d**
A.3(E)	determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d (S)

Stimulus

Word Problem	Verbal Description*	Chart/Table	Graph*
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

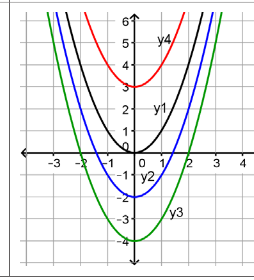
Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop* (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Content Builder (see Appendix for Tree Diagram)

- Determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by:
  - $a - f(x)$
  - $f(x) + d$
  - $f(x - c)$
  - $f(bx)$

Instructional Implications

Students are expected to determine the effects on the graph of the quadratic parent function, $f(x) = x^2$, when specific constants and coefficients are used to transform the graph. Instruction should include opportunities for students to explore these transformations using technology, where the students graph several related functions and are asked to explain the connections between the functions and the graphs.

Functions	Graphs	Explanation
$y_1 = x^2$ $y_2 = x^2 - 2$ $y_3 = x^2 - 4$ $y_4 = x^2 + 3$		Adding or subtracting numbers to the x^2 term translates the function up or down.

After such explorations, students should be able to summarize the different types of transformations on the parent function $f(x) = x^2$. For example:

Change	Transformation
$f(x) = a \bullet x^2$	The parent function will be vertically stretched (appear "taller") when $a > 1$ or vertically compressed (appear "shorter") when a is between 0 and 1. The parent function will be reflected over the x-axis when a is negative ($a < 0$).
$f(x) = x^2 + d$	The parent function will be translated d units up (when $d > 0$) or down (when $d < 0$)
$f(x) = (x - c)^2$	The parent function will be translated c units to the right (when $c > 0$) or to the left (when $c < 0$).
$f(x) = (bx)^2$	The parent function will be horizontally stretched (appear "wider") when b is between 0 and 1 or horizontally compressed (appear "narrower") when $b > 1$. The parent function will be reflected over the y-axis when $b < 0$; however, because of the function's symmetry, this has no visual effect on the graph.

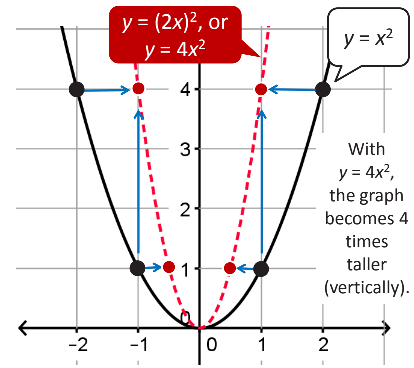
(continued)

* Used on STAAR

Instructional Implications (continued)

For translations, adding or subtracting constants inside of the squared term (like *c*) will affect the function horizontally (left or right), while constants added outside of the squared term (like *d*) will affect the function vertically (up or down). The same horizontal and vertical separation is also true for factors (like *a* and *b*) that create stretches and compressions; however, these are more difficult to differentiate.

In the example at right, consider the functions $y = (2x)^2$ ($f(x) = (bx)^2$ when $b = 2$) and $y = 4x^2$ ($f(x) = ax^2$ when $a = 4$). Note that the functions are algebraically equivalent since $(2x)^2 = 4x^2$. Likewise, the different transformations have equivalent results: a horizontal compression (where $b = 2$ makes the graph narrower) has the same effect as a vertical stretch (where $a = 4$ makes the graph taller).



With $y = (2x)^2$, the graph compresses ½ the distance toward the *y*-axis (horizontally).

Learning from Mistakes

Students may make the following mistakes:

- Confusing the signs for the values for *c* and *d* (positive or negative) and the direction of the translations (up, down, left, or right)*
- Confusing vertical stretch, vertical compression, horizontal stretch, and horizontal compression and their effects on the graph*
- Confusing translations (adding/subtracting numbers to the parent function *(f)x*) with stretches/compressions (multiplying of numbers to the parent function)*

Academic Vocabulary

downward*
horizontal shift*
parent function*
upward*
vertical shift*
width*

Interesting Items

A.7(C) 2024 #21
A.7(C) 2021 #28
A.7(C) 2018 #48
A.7(C) 2017 #24

A.12(A)

A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

(A) decide whether relations represented verbally, tabularly, graphically, and symbolically define a function

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

domain
range

Interesting Items

N/A

Role in Concept Development

Supports

- A.6(A) determine the domain and range of quadratic functions and represent the domain and range using inequalities
- A.7(A) graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry

Connection/ Relevance

Much of Algebra I is based on the identification of functions and functional relationships. Students must be able to identify the relationship between the domain and range of quadratic functions in order to appropriately write, solve, analyze, and evaluate functions.

When to Teach

Before/Prerequisite to A.6(A) and A.7(A)

Instructional Implications

Instruction should include defining a function as a relation where each element in the domain is paired with exactly one element in the range, then applying this definition in multiple representations.

First, students should be able to identify functions from a table of *x*- and *y*-values. If a relation is a function, then no *x*-value will be repeated in the table.

This is NOT a function:

x	y
5	1
6	2
7	3
6	4

(An *x*-value repeats; or, for $x = 6$, there are two possible *y*-values.)

This IS a function:

x	y
1	5
2	6
3	5
4	2

(Even though *y*-values may repeat, no *x*-value repeats; or, there is only one *y*-value for each *x*.)

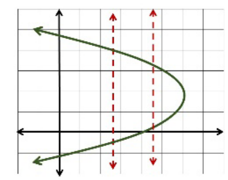
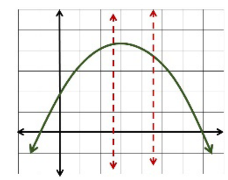
(continued)

* Used on STAAR

Role in Concept Development (continued)

Instructional Implications

Students should also be able to recognize a function from the graph. On the coordinate plane, functions will pass the “vertical line test,” which means that any vertical line will intersect the graph of a function at no more than one point.

If the graph of a relation has two points that are “stacked” atop one another, the relation is not a function (and will fail the vertical line test).	Parabolas that open to the left or right are NOT functions.  (A vertical line crosses the graph at more than one point.)	Parabolas that open up or down ARE functions.  (Any vertical line will only cross the graph once.)

Symbolically, relations are functions if they can be written as a single equation in “$y =$” form. In the cluster for quadratic functions, an equation such as $4x^2 + y = 5$ is a function because it can be rewritten as $y = -4x^2 + 5$. However, a relation such as $x + y^2 = 4$ (with a "y^2" term) is not a function, because it cannot be written as a single equation in “$y =$” form.

Students should be able to identify whether a relation is a function from a verbal description, by applying the definition to the *x*- and *y*-variables represented in the situation. For example:

Description	In a high school, each student’s grade level (x) is paired with the student’s height (y) in inches.	In a high school, each student’s ID number (x) is paired with the student’s grade level (y).
Is it a function?	NO	YES
Reason	Many students will be in the same grade but have different heights. So, the same *x*-value will be paired with many different *y*-values.	Each student has a unique ID number that will not be repeated, and a student will not be classified in more than one grade level.

Learning from Mistakes

Students may make the following mistakes:

- Switching *x* and *y* values (or switching domain and range)
- Confusing the rule about repeated *y*-values (which can be a function) with repeated *x*-values (which is not a function)

A.12(B)

A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

(B) evaluate functions, expressed in function notation, given one or more elements in their domains

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

domain
function notation
range

Interesting Items

N/A

Role in Concept Development

Supports

- A.6(A) determine the domain and range of quadratic functions and represent the domain and range using inequalities
- A.8(A) solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula

Connection/ Relevance

Understanding function notation is important for high school mathematics both symbolically (for writing equations and functions) and conceptually (to understand the input/output relationship between the domain and range of functions).

When to Teach

Before/Prerequisite to A.6(A) and A.8(A)

Instructional Implications

Instruction should include the use of function notation to describe the process of evaluating a function with values from its domain. For example, consider the function $f(x) = 2x^2 – 3x$. When given an expression such as $f(4)$, students should know that this means to evaluate the function $f(x)$ when $x = 4$ or substitute $x = 4$ into the expression. Here, $f(4) = 2(4)^2 - 3(4) = 32 – 12 = 20$. Students could summarize by writing $f(4) = 20$ (read as "*f* of 4 equals 20"). Students should recognize that $f(4) = 20$ indicates that 4 is in the domain of the function and is paired with 20 in the range.

Students may also recognize that function notation provides a more efficient way of writing problems with x's and y's. For example, see the comparison chart below.

Notation	Function notation	"$y =$" notation
Example	$f(x) = 2x^2 – 3x$	$y = 2x^2 – 3x$
Item	Find $f(4)$.	Find y when $x = 4$.
Answer	$f(4) = 20$	When $x = 4$, $y = 20$.

Learning from Mistakes

Students may make the following mistakes:

- Making sign errors or arithmetic mistakes in evaluating expressions
- After evaluating a function, thinking that this requires "solving for *f*." (e.g., given the expression $f(4) = 20$, mistakenly thinking they need to divide both sides by 4 to "solve for *f*."

* Used on STAAR