TEKS Cluster: Simplifying Expressions

A.10 **Number and algebraic methods.** The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions.

A.11 **Number and algebraic methods.** The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms.

Exponents

Readiness Standards

A.11(B) simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents

Polynomials

Readiness Standards

A.10(E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two

Supporting Standards

A.10(A) add and subtract polynomials of degree one and degree two
A.10(B) multiply polynomials of degree one and degree two
A.10(C) determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend
A.10(D) rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property
A.10(F) decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite the binomial

Radicals

Readiness Standards

A.11(B) simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents

Supporting Standards

A.11(A) simplify numerical radical expressions involving square roots

A.11(B) Readiness (pg. 1 of 2)

Subcluster: Exponents

TEKS Scaffold

TEKS	Student Expectation
2A.7(H)	solve equations involving rational exponents (R)
A.11(B)	**A.11 Number and algebraic methods.** The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to: **(B) simplify numeric and algebraic expressions using the laws of exponents, including integral** and rational **exponents**
6.7(A)	generate equivalent numerical expressions using order of operations, including whole number exponents and prime factorization (R)

Stimulus

Word Problem*	Verbal Description*	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop* (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Content Builder (see Appendix for Tree Diagram)

- Simplify numerical expressions using the laws of exponents, including integral exponents
- Simplify algebraic expressions using the laws of exponents, including integral exponents

Instructional Implications

Students are expected to simplify numeric and algebraic expressions using the laws of exponents. Exponents should be both positive and negative integers. Instruction should include development of the rules (laws) for products, quotients, and powers of terms with like bases.

$b^x \cdot b^y = b^{x+y}$	To multiply two powers of the same base, add the exponents.
$\frac{b^x}{b^y} = b^{x-y}$	To divide two powers of the same base, subtract the exponents.
$\left(b^x\right)^y = b^{xy}$	To raise a power to another power, multiply the exponents.

Initially, examples may include simple, whole-number exponents. For example:

$$x^3 \cdot x^7 = x^{3+7} = x^{10} \qquad \frac{m^5}{m^3} = m^{5-3} = m^2 \qquad \left(r^3\right)^5 = r^{3\cdot5} = r^{15}$$

These laws can then be used to derive the meaning of zero exponents and negative integer exponents.

Easy example: $\frac{6^{20}}{6^8} = 6^{20-8} = 6^{12}$	Easy example: $\frac{6^4}{6^3} = 6^{4-3} = 6^1 = 6$
Related example: $\frac{6^{20}}{6^{20}} = 6^{20-20} = 6^0$ So what is 6^0?	Related example: $\frac{6^3}{6^4} = 6^{3-4} = 6^{-1}$ So what is 6^{-1}?
Figure out: $\frac{6^{20}}{6^{20}} = 1$, so $6^0 = 1$	Figure out: $\frac{6^3}{6^4} = \frac{6\cdot6\cdot6}{6\cdot6\cdot6\cdot6} = \frac{1}{6} = 6^{-1}$
Definition: $n^0 = 1$ (when $n \neq 0$) Any non-zero value to the zero power equals 1.	Definition: $x^{-p} = \frac{1}{x^p}$ Negative exponents indicate reciprocals.

(continued)

* Used on STAAR

Instructional Implications (continued)

Ultimately, students should be able to combine these rules to simplify expressions such as those included below.

1. Simplify $\frac{a^3b^5c^9}{a^8b^3c^5}$, and rewrite using only positive exponents.

   Sample solution: $\frac{a^3b^5c^9}{a^8b^3c^5} = a^{-5}b^2c^4 = \frac{b^2c^4}{a^5}$

2. Which expression is equivalent to $2^{12} + 2^{12}$?

   A) 2^{13} B) 2^{24} C) 4^{12} D) 4^{24}

   Solution: A, because $2^{12} + 2^{12} = 2(2^{12}) = 2^1 \cdot 2^{12} = 2^{13}$

Learning from Mistakes

Students may make the following mistakes:

- Confusing the rules for multiplying terms (adding exponents) and raising terms to powers (multiplying exponents)*
- Thinking that negative exponents generate negative values (instead of fractions or reciprocals)*
- Confusing the rules for multiplying exponential terms (adding exponents) with the rules for dividing exponential terms (subtracting exponents)
- Confusing the rules for coefficients and exponents when multiplying terms or raising terms to powers (e.g., $(3c^6)(5c^3) = 8c^9$ instead of $15c^9$; or $(5m^3)^4 = 20m^{12}$ instead of $625m^{12}$)

Academic Vocabulary

integral exponent
laws of exponents

Interesting Items

A.11(B) 2024 #6
A.11(B) 2023 #35
A.11(B) 2018 #6
A.11(B) 2018 #28
A.11(B) 2018 #49
A.11(B) 2017 #6

* Used on STAAR

A.10(E) Readiness

Subcluster: Polynomials

TEKS Scaffold

TEKS	Student Expectation
2A.7(E)	determine linear and quadratic factors of a polynomial expression of degree three and of degree four, including factoring the sum and difference of two cubes and factoring by grouping (R)
A.10(E)	**A.10 Number and algebraic methods.** The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to: **(E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two**
A.10(B)	multiply polynomials of degree one and degree two (S)

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect* (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Content Builder (see Appendix for Tree Diagram)

- Factor trinomials in the form $ax^2 + bx + c$
- Factor perfect square trinomials of degree two

Instructional Implications

Students are expected to factor, if possible, trinomials in the form $ax^2 + bx + c$. Generally, this means to rewrite the trinomial as the product of two linear binomials (e.g., the expression $3x^2 + 10x – 8$ can be factored as $(3x – 2)(x + 4)$). The factors can be checked by multiplying the binomials using distribution ("FOIL") and combining like terms.

Instruction on factoring should begin with cases of $ax^2 + bx + c$ where $a = 1$ ($x^2 + bx + c$). Factoring can be accomplished by finding two numbers with a product of c but with a sum of b. In the expression $x^2 + 1x – 6$, $b = 1$ and $c = -6$. To factor the expression, students find two numbers which add up to 1 that will multiply to get an answer of -6 (3 and -2); so the factors are $(x + 3)$ and $(x – 2)$, and the original expression can be rewritten as $(x + 3)(x – 2)$.

Finding numbers with a given product and sum can be expanded to factor more difficult trinomials where $a \neq 1$. Instruction can include various methods for factoring, including strategies that expand the bx term to factor by grouping.

Instruction should also emphasize perfect square trinomials as a special case for factoring. When the two factors of a trinomial are the same, they can be rewritten as a binomial squared (e.g., $4x^2 – 20x + 25 = (2x – 5)(2x – 5) = (2x – 5)^2$). From repeated examples, students may generalize the pattern that perfect square trinomials of the form $ax^2 + bx + c$ have square numbers for a and c and a value of b that is twice the product of their square roots (e.g., $4x^2 – 20x + 25$; $\sqrt{4} = 2$ and $\sqrt{25} = 5$; $2(2 \bullet 5) = 20$).

Learning from Mistakes

Students may make the following mistakes:

- Making errors in distribution or combining like terms when checking the factors of a trinomial (especially with minus signs and/or negative numbers)*
- Thinking you can "distribute" the exponent to both terms in a binomial that is being squared. For example, students may incorrectly rewrite $(2x + 3)^2$ as $(2x)^2 + (3)^2 = 4x^2 + 9$, or students may incorrectly factor $16y^2 – 25$ as $(4y – 5)^2$.
- "Factoring" polynomials that are prime
- Not completely factoring a polynomial to its simplest form. For example, students may factor $8x^2 + 4x – 12$ as $(4x + 6)(2x – 2)$ rather than $2(2x + 3)(x – 1)$.

Academic Vocabulary

factor*
perfect square trinomial
trinomial

Interesting Items

A.10(E) 2022 #33
A.10(E) 2018 #31

* Used on STAAR

A.10(A) Supporting

Subcluster: Polynomials

A.10(A)

A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:

(A) add and subtract polynomials of degree one and degree two

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

binomial
degree (of a polynomial)
equivalent*
like terms
monomial
polynomial
quadratic
trinomial

Interesting Items

A.10(A) 2024 #35
A.10(A) 2017 #13

Role in Concept Development

Supports	A.10(E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two
Connection/ Relevance	Combining like terms is foundational to being able to factor and solve quadratic expressions.
When to Teach	Before/Prerequisite to A.10(E)
Instructional Implications	Instruction should include adding and subtracting polynomials of degree one and degree two. Students should be able to identify and combine like terms (terms with the same variable(s) raised to the same power). To combine like terms, students must add or subtract their coefficients. $(2x^2 + 3x – 9) + (-7x + 4)$ In the problem above, students must add a quadratic trinomial and a linear binomial. Students should be able to identify that $3x$ and $-7x$ are like terms, as well as -9 and 4. Adding the polynomials together yields $2x^2 – 4x – 5$. Subtracting polynomials follows the same rules. However, students must remember to add the opposite of second polynomial (distribute the minus sign over the parentheses). $(5x^2 + x + 8) – (3x^2 + 5x – 3)$ For example, the subtraction problem above can be rewritten as: $5x^2 + x + 8 + (-3x^2) – 5x + 3$ Combining like terms yields an answer of $2x^2 – 4x + 11$.
Learning from Mistakes	Students may make the following mistakes: • Making errors in combining terms, especially with positive/negative coefficients • When subtracting polynomials, forgetting to distribute the negative sign to every term in the second expression

A.10(B) Supporting

Subcluster: Polynomials

A.10(B)

A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:

(B) multiply polynomials of degree one and degree two

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

- binomial
- degree (of a polynomial)
- distribute
- like terms
- monomial
- polynomial
- quadratic
- trinomial

Interesting Items

A.10(B) 2018 #36
A.10(B) 2016 #54

Role in Concept Development

Supports

A.10(E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two

Connection/ Relevance

Multiplying polynomials is foundational to being able to factor and solve quadratic expressions.

When to Teach

Before/Prerequisite to A.10(E)

Instructional Implications

Instruction should include multiplying polynomials of degree one and degree two. Students should know how to multiply monomials in the same variable by multiplying coefficients and adding exponents (e.g., $(5a^2)(3a^1) = 15a^3$). Students should know how to use this skill with the distributive property, as shown in the examples below.

A) $3x(4x – 2) = 3x(4x) + (3x)(-2)$ $= 12x^2 – 6x$	Distribute the 3x to both 4x and -2.
B) $-2x^2(7x – 3) = (-2x^2)(7x) + (-2x^2)(-3)$ $= -14x^3 + 6x^2$	Distribute the $-2x^2$ to both 7x and -3.
C) $(2x – 1)(4x + 3) = (2x – 1)(4x) + (2x – 1)(3)$ $= (2x)(4x) + (-1)(4x) + (2x)(3) + (-1)(3)$ $= 8x^2 – 4x + 6x – 3$ $= 8x^2 + 2x – 3$	Distribute, then distribute again. Combine like terms.

When multiplying binomials (example C above), the double-distribution process is known by the acronym “FOIL” (First, Outside, Inside, Last). When multiplying polynomials with more than two terms, students mush understand that the distributive process results in every term in the first factor being multiplied by every term in the second factor. For example:

$$\overset{\text{binomial}}{(2x + 5)}\overset{\text{trinomial}}{(3x^2 - 4x + 7)}$$

$$= \mathbf{2x(3x^2 - 4x + 7) + 5(3x^2 - 4x + 7)}$$

$$= \underbrace{(2x)(3x^2) + (2x)(-4x) + (2x)(7)}_{\text{(2x times each term in the trinomial}} + \underbrace{(5)(3x^2) + (5)(-4x) + (5)(7)}_{\text{(5 times each term in the trinomial)}}$$

$$\begin{aligned} &= 6x^3 + (-8x^2) + 14x + 15x^2 + (-20x) + 35 \\ &= 6x^3 + (-8x^2 + 15x^2) + (14x - 20x) + 35 \\ &= 6x^3 + 7x^2 - 6x + 35 \end{aligned}$$

Rearranging to combine like terms

Learning from Mistakes

Students may make the following mistakes:

- Making errors in combining terms, especially with positive/negative coefficients
- When multiplying polynomials, forgetting to distribute to every term in each expression
- Failing to distribute a negative sign when subtracting terms grouped in parentheses

* Used on STAAR

A.10(C) Supporting

Subcluster: Polynomials

A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:

A.10(C) **(C) determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend**

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

dividend
divisor
polynomial
quotient

Interesting Items

A.10(C) 2023 #16

Role in Concept Development

Supports

A.10(E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two

Connection/ Relevance

Determining the quotient of two polynomials is closely related to multiplying two binomials. To begin, students may compare division to finding a missing factor in a polynomial multiplication problem, such as $x^2 + 6x + 8 = (x + 2) \bullet (?)$. Here, the missing factor can be described as a quotient: $(x^2 + 6x + 8) \div (x + 2)$.

When to Teach

After A.10(E)

Instructional Implications

Instruction should include division of linear (first degree) and quadratic (second degree) polynomials. To determine the quotient, students can initially rely on factoring, but more challenging items may require polynomial long division.

As when dividing whole numbers, long division of polynomials requires a process of dividing, multiplying, subtracting, and bringing down (then repeating). For example:

	Item	Work	Answer
Example	$\frac{171}{7}$	24 $7\overline{)171}$ -14 31 -28 3	$24\frac{3}{7}$
A)	$\frac{6x^2-5x-21}{3x-7}$	$2x+3$ $3x-7\overline{)6x^2-5x-21}$ $-(6x^2-14x)$ $9x-21$ $-(9x-21)$ 0	$2x+3$
B)	$\frac{2x^2+9x+4}{x+3}$	$2x+3$ $3x+3\overline{)2x^2+9x+4}$ $-(2x^2+6x)$ $3x+4$ $-(3x-9)$ -5	$2x+3-\frac{5}{x+3}$

Note that some quotients have a non-zero “remainder” (item B). In these cases, the answer can be written as the quotient plus the remainder over the divisor, much like a mixed number in a problem involving whole number division.

Learning from Mistakes

Students may make the following mistakes:
- Making errors in combining terms, especially with positive/negative coefficients
- When dividing polynomials, forgetting to distribute the negative sign when subtracting

* Used on STAAR

A.10(D) Supporting

Subcluster: Polynomials

A.10(D)

A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:

(D) rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

binomial
degree (of a polynomial)
distribute
equivalent*
factor
greatest common factor
like terms
monomial
polynomial
quadratic
trinomial

Interesting Items

A.10(D) 2023 #19
A.10(D) 2016 #15

Role in Concept Development

Supports

A.10(E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two

Connection/ Relevance

Effectively using the distributive property to generate equivalent polynomial expressions is foundational to factoring and solving quadratic expressions.

When to Teach

Before/Prerequisite to A.10(E)

Instructional Implications

Instruction should include rewriting polynomials of degree one and degree two in equivalent forms using the distributive property. The distributive property states that multiplying a factor by a sum is equivalent to multiplying the factor by each term in the sum (e.g., $A(x + y + z) = Ax + Ay + Az$). Students should know how to apply this skill with the terms in both linear and quadratic polynomials.
For example:

A) $3(4x - 2) = 3(4x) + (3)(-2)$ $= 12x - 6$	Distribute the 3 to both $4x$ and -2.
B) $-2x(7x - 3) = (-2x)(7x) + (-2x)(-3)$ $= -14x^2 + 6x$	Distribute the $-2x$ to both $7x$ and -3.
C) $-6(2x^2 + x - 5) = (-6)(2x^2) + (-6)(x) + (-6)(-5)$ $= -12x^2 - 6x + 30$	Distribute the -6 to each term in the trinomial.

In addition to multiplying polynomials, the distributive property can also be used “backwards” to introduce factoring. For this skill, students must rewrite a polynomial as the product of another polynomial and a monomial (common factor). Instruction can begin with fill-in-the-blank samples. For example:

Sample	Answer(s)
$15m - 24 = 3(\_\_\_ - \_\_\_)$	$5m$ and 8
$4z^2 - 24z = \_\_\_(2z - 12)$	$2z$
$-60x^2 + 24x - 30 = \_\_\_(\_\_\_ - 4x + \_\_\_)$	-6, $10x^2$, and 5

Learning from Mistakes

Students may make the following mistakes:

- When multiplying polynomials, forgetting to distribute to every term in the parenthetical expression*

A.10(F) Supporting

Subcluster: Polynomials

A.10(F)

A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:

> **(F) decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite the binomial**

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop* (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

binomial
factor*
square/perfect square

Interesting Items

A.10(F) 2022 #43

Role in Concept Development

Supports

A.10(E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two

Connection/ Relevance

This supporting standard describes a specific case when the polynomial is a binomial difference of two squares. This skill is foundational to factoring all types of polynomials.

When to Teach

With – context

Instructional Implications

Instruction should include factoring polynomials that are written as the difference of two squares. Students should be able to identify a “difference of two squares” as a binomial with a minus sign between two terms that are each perfect squares. For example:

Polynomial	Difference of Two Squares?	Reason
$4x^2 – 25x – 49$	NO	Not a binomial (has 3 terms instead of 2)
$4x^2 + 25$	NO	Not a difference (has a plus sign)
$4x^2 – 30$	NO	30 is not a perfect square.
$4x^2 – 25$	YES	A difference (–), two terms, both are perfect squares.

Students should explore multiplying binomials that may or may not result in a difference of two squares. For example:

Item	Answer	Difference of Two Squares?
$(3x + 5)(3x – 5)$	$9x^2 – 25$	YES
$(2x – 7)(2x – 7)$	$4x^2 – 14x + 49$	NO
$(2x – 7)(2x + 7)$	$4x^2 – 49$	YES

From samples such as these, students can develop a general rule for factoring a difference of two squares, such as $a^2 – b^2 = (a + b)(a – b)$. Students should then be able to rewrite polynomials of this form. For example:

Polynomial	Rewrite
$9c^2 – 100$	$(3c + 10)(3c – 10)$
$36x^2 – 49y^2$	$(6x + 7y)(6x – 7y)$
$4m^2 + 121m$	Not a difference of two squares

Learning from Mistakes

Students may make the following mistakes:

- Incorrectly applying the "difference of two squares" rule to include binomials that are sums or to include terms that are not squares

* Used on STAAR

A.11(B) Readiness (pg. 1 of 2)

Subcluster: Radicals

TEKS Scaffold

TEKS	Student Expectation
2A.7(H)	solve equations involving rational exponents (R)
A.11(B)	**A.11 Number and algebraic methods.** The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to: **(B) simplify numeric and algebraic expressions using the laws of exponents, including** integral and **rational exponents**
6.7(A)	generate equivalent numerical expressions using order of operations, including whole number exponents and prime factorization (R)

Stimulus

Word Problem*	Verbal Description*	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Content Builder (see Appendix for Tree Diagram)

- Simplify numerical expressions using the laws of exponents, including rational exponents
- Simplify algebraic expressions using the laws of exponents, including rational exponents

Instructional Implications

Students are expected to simplify numeric and algebraic expressions using the laws of exponents. Instruction should include development of the rules (laws) for products, quotients, and powers of terms with like bases.

Initially, examples may include whole-number exponents; but the same rules can also be applied to rational exponents. For example:

$b^x \cdot b^y = b^{x+y}$	To multiply two powers of the same base, add the exponents.
$\frac{b^x}{b^y} = b^{x-y}$	To divide two powers of the same base, subtract the exponents.
$\left(b^x\right)^y = b^{xy}$	To raise a power to another power, multiply the exponents.

$$x^3 \cdot x^7 = x^{3+7} = x^{10} \qquad \frac{m^5}{m^3} = m^{5-3} = m^2 \qquad \left(r^3\right)^5 = r^{3\cdot5} = r^{15}$$

$$x^{\frac{2}{3}} \cdot x^{\frac{4}{3}} = x^{\frac{2}{3}+\frac{4}{3}} = x^{\frac{6}{3}} = x^2 \qquad \frac{m^{\frac{1}{2}}}{m} = m^{\frac{1}{2}-1} = m^{-\frac{1}{2}} \qquad (r^4)^{\frac{3}{2}} = r^{4\cdot\frac{3}{2}} = r^6$$

These laws can then be used to derive the meaning of rational number exponents.

Easy example: $(25)^3 \cdot (25)^7 = (25)^{3+7} = (25)^{10}$	Easy example: $((8)^3)^4 = (8)^{3\cdot4} = (8)^{12}$	Easy example: $\frac{8^7}{8^4} = 8^{7-4} = 8^3$
Related example: $(25)^{\frac{1}{2}} \cdot (25)^{\frac{1}{2}} = (25)^{\frac{1}{2}+\frac{1}{2}}$ $= (25)^1 = 25$ So what is $(25)^{\frac{1}{2}}$?	Related example: $\left((8)^{\frac{1}{3}}\right)^3 = (8)^{\frac{1}{3}\cdot3}$ $= (8)^1 = 8$ So what is $(8)^{\frac{1}{3}}$?	Related example: $\frac{8}{(8)^{\frac{1}{3}}} = (8)^{1-\frac{1}{3}} = (8)^{\frac{2}{3}}$ So what is $(8)^{\frac{2}{3}}$?
Figure out: $(25)^{\frac{1}{2}} = \sqrt{25} = 5$	Figure out: $(8)^{\frac{1}{3}} = \sqrt[3]{8} = 2$	Figure out: $(8)^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)\left(\sqrt[3]{8}\right) = \left(\sqrt[3]{8}\right)^2 = 4$
Definition: $(x)^{\frac{1}{2}} = \sqrt{x}$ Rational powers mean roots (square roots, cube roots).	Definition: $(x)^{\frac{1}{n}} = \sqrt[n]{x}$ A power of $\frac{1}{n}$ indicates an "nth" root (cube roots, fourth roots, etc.)	Definition: $(x)^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$ Rational exponents indicate roots raised to powers.

(continued)

* Used on STAAR

Instructional Implications (continued)

Ultimately, students should be able to combine these rules to simplify expressions such as those included below.

1. Simplify: $\left(\left(a^{\frac{1}{2}}b^{\frac{2}{3}}\right)\cdot\left(a^{\frac{7}{2}}b^{\frac{4}{3}}\right)\right)^{\frac{1}{2}}$

   Sample solution: $\left(\left(a^{\frac{1}{2}+\frac{7}{2}}\right)\cdot\left(b^{\frac{2}{3}+\frac{4}{3}}\right)\right)^{\frac{1}{2}}=\left(a^4b^2\right)^{\frac{1}{2}}=a^2b$

2. Rewrite $\sqrt{x^3\cdot y}\cdot\sqrt[4]{x^2\cdot y^3}$ with rational exponents, then simplify.

   Sample solution: $(x^3\cdot y)^{\frac{1}{2}}\cdot(x^2\cdot y^3)^{\frac{1}{4}}=x^{\frac{3}{2}}\cdot y^{\frac{1}{2}}\cdot x^{\frac{2}{4}}\cdot y^{\frac{3}{4}}=x^2\cdot y^{\frac{5}{4}}$

Learning from Mistakes

Students may make the following mistakes:

- Confusing the rules for multiplying terms (adding exponents) and raising terms to powers (multiplying exponents)*
- Thinking that negative exponents generate negative values (instead of fractions or reciprocals)*
- Confusing the rules for multiplying exponential terms (adding exponents) with the rules for dividing exponential terms (subtracting exponents)
- Confusing the rules for coefficients and exponents when multiplying terms [e.g., $(3c^6)(5c^3) = 8c^9$ instead of $15c^9$; or $(5m^3)^4 = 20m^{12}$ instead of $625m^{12}$]
- Confusing the rules for coefficients and exponents when raising terms to powers [e.g., $(64m^4)^{\frac{1}{2}} = 32m^2$ instead of $8m^2$]

Academic Vocabulary

laws of exponents
rational exponent

Interesting Items

A.11(B) 2018 #6
A.11(B) 2018 #28
A.11(B) 2018 #49
A.11(B) 2017 #6

* Used on STAAR

A.11(A) Supporting

Subcluster: Radicals

A.11(A)

A.11 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to:

(A) simplify numerical radical expressions involving square roots

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Item Types

Multiselect (2 pts)	Match Table Grid (2 pts)	Drag and Drop (1-2 pts)	Fraction Model (1-2 pts)
Hot Spot (1-2 pts)	Inline Choice (1-2 pts)	Number Line (1-2 pts)	Graphing (1-2 pts)
Text Entry (1-2 pts)	Equation Editor (1-2 pts)	Multiple Choice* (1 pt)	

Academic Vocabulary

factor
perfect square
radical
rationalizing the denominator
simplified/simplest radical form
square root

Interesting Items

N/A

Role in Concept Development

Supports

- A.8(A) solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula
- A.11(B) simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents

Connection/ Relevance

Students need to recognize that radical expressions can be written in different but equivalent forms. This will be critical when solving quadratic equations in which answers can be left in radical form (with square roots).

When to Teach

- With A.11(B)
- With or after A.8(A)

Instructional Implications

Much like simplifying fractions, simplifying radicals involves breaking down numbers into their factors to rewrite expressions in a way that's easier to work with. Students need to understand the rule for multiplying radical expressions: $\sqrt{a} \bullet \sqrt{b} + \sqrt{ab}$ (e.g., $\sqrt{2} \bullet \sqrt{5} = \sqrt{10}$ and $\sqrt{3} \bullet \sqrt{7} = \sqrt{21}$). Sometimes, the product of two radical expressions can be simplified further (e.g., $\sqrt{2} \bullet \sqrt{18} = \sqrt{36}$ or $\sqrt{7} \bullet \sqrt{7} = \sqrt{49}$).

Expression	Work	Simplified Radical Form
$\sqrt{50}$	$\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$	$5\sqrt{2}$
$\sqrt{48}$	$\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$	$4\sqrt{3}$

In addition to finding perfect square factors, other methods can be used to simplify numerical radical expressions, such as the use of factor trees or prime factorization. Regardless of the method, students should be able to use simplifying radicals to recognize equivalent answers to algebra problems, such as solutions to quadratic equations.

For example, when solving an equation such as $x^2 - 8 = 10$, a student may correctly arrive at the answer $x = \pm \sqrt{18}$; however, an answer choice or test key may list the answer as $x = \pm 3\sqrt{2}$. Students should be able to recognize that these expressions are equivalent; the latter is just in simplified radical form.

Learning from Mistakes

Students may make the following mistakes:

- Simplifying with factors that are not square numbers
- Incorrectly identifying square roots*

* Used on STAAR