

TEKS Cluster: Systems of Equations and Inequalities

- A.2 Linear functions, equations, and inequalities.** The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.
- A.3 Linear functions, equations, and inequalities.** The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.
- A.5 Linear functions, equations, and inequalities.** The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions.

Systems of Equations

Readiness Standards

- A.2(I) write systems of two linear equations given a table of values, a graph, and a verbal description
A.5(C) solve systems of two linear equations with two variables for mathematical and real-world problems

Supporting Standards

- A.3(F) graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist
A.3(G) estimate graphically the solutions to systems of two linear equations with two variables in real-world problems

Inequalities

Readiness Standards

- A.3(D) graph the solution set of linear inequalities in two variables on the coordinate plane

Supporting Standards

- A.2(H) write linear inequalities in two variables given a table of values, a graph, and a verbal description
A.3(H) graph the solution set of systems of two linear inequalities in two variables on the coordinate plane
A.5(B) solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides

TEKS Scaffold

| TEKS | Student Expectation |
|---------|--|
| 2A.3(A) | formulate systems of equations, including systems consisting of three linear equations in three variables and systems consisting of two equations, the first linear and the second quadratic (R) |

A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

A.2(I)

(I) write systems of two linear equations given a table of values, a graph, and a verbal description

| | |
|--------|---|
| 8.9(A) | identify and verify the values of x and y that simultaneously satisfy two linear equations in the form $y = mx + b$ from the intersections of the graphed equations (S) |
|--------|---|

Stimulus

| Word Problem* | Verbal Description | Chart/Table* | Graph* |
|----------------------|--------------------|---------------|-------------------|
| Equation/Expression* | Manipulatives | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

Item Types

| | | | |
|----------------------|---------------------------|-------------------------|--------------------------|
| Multiselect (2 pts) | Match Table Grid (2 pts) | Drag and Drop (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

Content Builder (see Appendix for Tree Diagram)

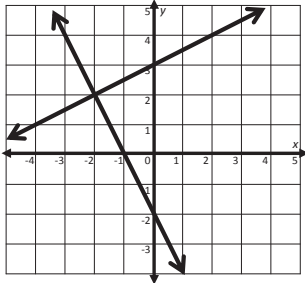
- Write systems of two linear equations given a table of values
- Write systems of two linear equations given a graph
- Write systems of two linear equations given a verbal description

Instructional Implications

Students are expected to write systems of two linear equations from a variety of prompts. First, students must use a table of values to generate a system of equations. Instruction should include both real-world and purely mathematical examples. For example:

| Sample | System | Method | | | | | | | | | | | | | | | |
|--|--|--|---------------|-------|----|----|----|-----|-----|----|-----|-----|----|-----|-----|---|--|
| Two lines are graphed on a coordinate grid. Their x - and y - values are given in the table. | $y_1 = \begin{cases} x + 5 \\ 2x - 4 \end{cases}$ | From a table, writing each equation in the system consists of either recognizing the slope and y -intercept from the patterns in the table or using ordered pairs with the formula for slope and the point-slope equation for a line [see A.2(C)]. | | | | | | | | | | | | | | | |
| <table border="1"> <thead> <tr> <th>x</th> <th>y_1</th> <th>y_2</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>5</td> <td>-4</td> </tr> <tr> <td>1</td> <td>4</td> <td>-2</td> </tr> <tr> <td>2</td> <td>3</td> <td>0</td> </tr> <tr> <td>3</td> <td>2</td> <td>2</td> </tr> </tbody> </table> | x | | y_1 | y_2 | 0 | 5 | -4 | 1 | 4 | -2 | 2 | 3 | 0 | 3 | 2 | 2 | |
| x | y_1 | y_2 | | | | | | | | | | | | | | | |
| 0 | 5 | -4 | | | | | | | | | | | | | | | |
| 1 | 4 | -2 | | | | | | | | | | | | | | | |
| 2 | 3 | 0 | | | | | | | | | | | | | | | |
| 3 | 2 | 2 | | | | | | | | | | | | | | | |
| Two companies have different rates for screen-printing T-shirts for an upcoming event. | $\begin{cases} (A) y = 4x + 50 \\ (B) y = 5x + 10 \end{cases}$ | | | | | | | | | | | | | | | | |
| <table border="1"> <thead> <tr> <th># of T-shirts</th> <th>Cost at Co. A</th> <th>Cost at Co. B</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>50</td> <td>10</td> </tr> <tr> <td>20</td> <td>130</td> <td>110</td> </tr> <tr> <td>40</td> <td>210</td> <td>210</td> </tr> <tr> <td>60</td> <td>290</td> <td>310</td> </tr> </tbody> </table> | # of T-shirts | Cost at Co. A | Cost at Co. B | 0 | 50 | 10 | 20 | 130 | 110 | 40 | 210 | 210 | 60 | 290 | 310 | | |
| # of T-shirts | Cost at Co. A | Cost at Co. B | | | | | | | | | | | | | | | |
| 0 | 50 | 10 | | | | | | | | | | | | | | | |
| 20 | 130 | 110 | | | | | | | | | | | | | | | |
| 40 | 210 | 210 | | | | | | | | | | | | | | | |
| 60 | 290 | 310 | | | | | | | | | | | | | | | |

Students must also use a graph of two linear equations to write the related system.

| Sample | System | Method |
|---|---|--|
|  | $\begin{cases} y = -2x - 2 \\ y = \frac{1}{2}x + 3 \end{cases}$ | From a graph, writing each equation in the system consists of either recognizing the slope and y -intercept from the points and patterns on the graph or using ordered pairs from each line with the formula for slope and the point-slope equation for a line [see A.2(C)]. |

(continued)

Instructional Implications (continued)

Instruction should also include verbal descriptions of real-world situations from which students must write a system of linear equations. Steps in this process include identifying the variables, associating the variables with constants given in the description, and establishing the two facts that will form the two equations in the system. For example:

| | | |
|--------------------|---|--------------------------------------|
| Verbal description | Tickets to the basketball game cost \$5 for general admission and \$3 for students. At a recent game, a total of 286 tickets were sold, generating \$1,006 in revenue. How many of each type of ticket were sold? | |
| Variables | Let g = the number of general admission tickets sold, and s = the number of student tickets sold | |
| Constants | \$5 = cost for each general admission ticket (g) \$3 = cost for each student ticket (s) | |
| Facts | The total amount of money earned was \$1,006. | The total number of tickets was 286. |
| Equations | $5g + 3s = 1006$ | $g + s = 286$ |

Note: This standard [A.2(I)] relates only to writing systems. Solving systems is described in A.5(C).

Learning from Mistakes

Students may make the following mistakes:

- Only writing one equation instead of two when writing a system
- Switching values for x and y in the slope formula or in the point-slope form of a linear equation*
- Confusing the signs of a line’s slope or y -intercept (positive or negative)
- Given verbal descriptions and/or graphs, having trouble identifying the variables and confusing the constants and coefficients that go with each*

Academic Vocabulary

system of equations*

Interesting Items

- A.2(I) 2023 #37
- A.2(I) 2021 #24
- A.2(I) 2017 #48
- A.2(I) 2016 #13

TEKS Scaffold

| TEKS | Student Expectation |
|---------|--|
| 2A.3(B) | solve systems of three linear equations in three variables by using Gaussian elimination, technology with matrices, and substitution (R) |

A.5(C) **Linear functions, equations, and inequalities.** The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:

(C) solve systems of two linear equations with two variables for mathematical and real-world problems

| | |
|--------|--|
| 8.9(A) | identify and verify the values of x and y that simultaneously satisfy two linear equations in the form $y = mx + b$ from the intersections of the graphed equations (S) |
| 8.8(C) | model and solve one-variable equations with variables on both sides of the equal sign that represent mathematical and real-world problems using rational number coefficients and constants (R) |

Stimulus

| Word Problem* | Verbal Description | Chart/Table* | Graph |
|----------------------|--------------------|---------------|-------------------|
| Equation/Expression* | Manipulatives | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

Item Types

| | | | |
|----------------------|---------------------------|-------------------------|--------------------------|
| Multiselect (2 pts) | Match Table Grid (2 pts) | Drag and Drop (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

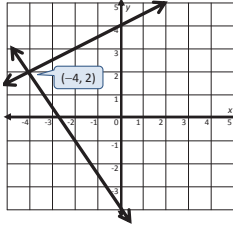
Content Builder (see Appendix for Tree Diagram)

- Solve systems of linear equations with two variables for mathematical problems
- Solve systems of linear equations with two variables for real-world problems

Instructional Implications

Students should solve systems of linear equations with two variables in a variety of problem situations. Instruction must include defining the solution to a linear system as the ordered pair (x, y) that satisfies both equations in the system. Students should also investigate the various methods for solving a system of equations, starting with graphing. By graphing both equations in a linear system on the same coordinate grid, students can identify the solution as the lines' point of intersection.

Other algebraic methods for solving systems include substitution and combination/elimination. In the substitution method, students solve one of the system's equations for a specific variable and substitute its value into the other equation. In the combination/elimination method, equations in a system can be added together or subtracted to cancel variables. In either case, the process results in a single-variable equation that can be solved algebraically for one of the values in the system. The other value in the solution can be found algebraically through a numeric substitution into one of the original equations.

| Method | Graphing | Substitution | Elimination |
|---------------|--|---|--|
| Sample system | $\begin{cases} y = \frac{1}{2}x + 4 \\ y = -\frac{3}{2}x - 4 \end{cases}$ | $\begin{aligned} y &= 3x - 1 \\ 4x - 2y &= 10 \end{aligned}$ | $\begin{aligned} 2x + 3y &= 1 \\ x - 3y &= 14 \end{aligned}$ |
| Work |  | $\begin{aligned} 4x - 2(3x - 1) &= 10 \\ 4x - 6x + 2 &= 10 \\ -2x + 2 &= 10 \\ -2x &= 8 \\ x &= -4 \\ y &= 3(-4) - 1 \\ y &= -13 \end{aligned}$ | $\begin{aligned} 2x + 3y &= 1 \\ + x - 3y &= 14 \\ \hline 3x &= 15 \\ x &= 5 \\ 2(5) + 3y &= 1 \\ 10 + 3y &= 1 \\ 3y &= -9 \\ y &= -3 \end{aligned}$ |
| Solution | $(-4, 2)$ | $(-4, -13)$ | $(5, -3)$ |

Learning from Mistakes

Students may make the following mistakes:

- Making arithmetic errors when solving equations*
- Combining equations that are not aligned at the equal sign
- Having difficulty with infinite solutions and "no solution" situations

Academic Vocabulary

infinite number*
system of equations*

Interesting Items

A.5(C) 2023 #40 A.5(C) 2018 #15
A.5(C) 2022 #49 A.5(C) 2016 #39
A.5(C) 2021 #50

A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(F) graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist

Role in Concept Development

Supports A.5(C) solve systems of two linear equations with two variables for mathematical and real-world problems

Connection/Relevance The graphing of systems of equations provides a visual for solving systems of equations.

When to Teach Before/Prerequisite to A.5(C)

Instructional Implications Instruction should include solving systems of two linear equations by graphing the lines on a coordinate grid. This may require rewriting the equations in slope-intercept form. Once the equations are graphed [see A.3(C)], the solution to the system can be identified as the point where the lines intersect.

Stimulus

| | | | |
|----------------------|---------------------|---------------|-------------------|
| Word Problem | Verbal Description* | Chart/Table | Graph* |
| Equation/Expression* | Manipulatives | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

Item Types

| | | | |
|----------------------|---------------------------|-------------------------|--------------------------|
| Multiselect (2 pts) | Match Table Grid (2 pts) | Drag and Drop (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

Academic Vocabulary

coincide
 intersect/intersection
 linear equation
 parallel
 solution*
 system of equations*

Interesting Items

A.3(F) 2022 #45
 A.3(F) 2017 #29

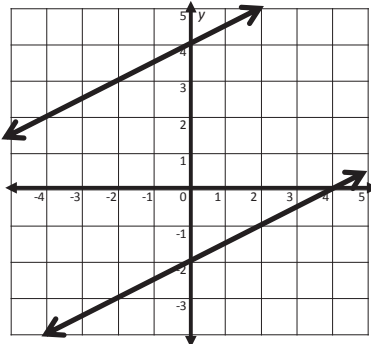
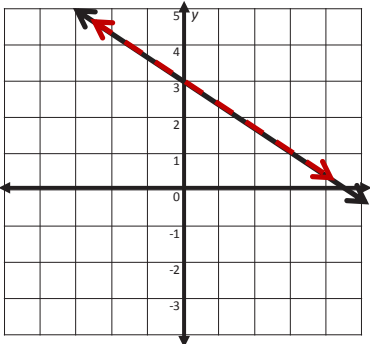
| | |
|----------------------|--|
| System | $x - 2y = -8$ $3x + 2y = -8$ |
| Slope intercept-form | $y = \frac{1}{2}x + 4$ $y = -\frac{3}{2}x - 4$ |
| Graph | |
| Solution | $(-4, 2)$ |

(continued)

Role in Concept Development (continued)

Instructional Implications

In some cases, the lines in a system may be parallel (same slope, but different y-intercepts), so there will be no point of intersection. When this occurs, students should identify the system as having no solution. In other cases, the two equations in a system may generate the exact same line (the linear equations coincide). When this occurs, students should recognize that the system has an infinite number of solutions, since every point on the line is a solution. For example:

| | | |
|----------------------|--|--|
| System | $x - 2y = -8$ $x - 2y = 4$ | $2x + 3y = 9$ $6y = -4x + 18$ |
| Slope intercept-form | $y = \frac{1}{2}x + 4$ $y = \frac{1}{2}x - 2$ | $y = -\frac{2}{3}x + 3$ $y = -\frac{2}{3}x + 3$ |
| Graph |  |  |
| Solution | No solution. (The lines are parallel.) | Every point on the line is a solution. (The lines coincide.) |

Learning from Mistakes

Students may make the following mistakes:

- When graphing, making errors with the slope and y-intercept by either switching the signs or positions*
- Confusing whether systems have no solutions or infinitely many*
- Identifying non-parallel line segments as having no solution as the visual does not physically show them intersecting

A.3(G) **A.3 Linear functions, equations, and inequalities.** The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(G) estimate graphically the solutions to systems of two linear equations with two variables in real-world problems

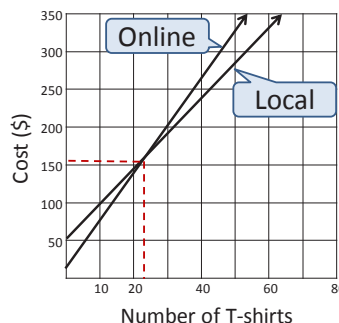
Role in Concept Development

Supports A.5(C) solve systems of two linear equations with two variables for mathematical and real-world problems.

Connection/Relevance Estimating solutions to systems of equations allows students to apply reasonableness when evaluating solutions to systems of linear equations.

When to Teach With A.5(C)

Instructional Implications Instruction should include using graphs to estimate solutions to systems of two linear equations in real-world problems. From a given graph, the solution to the system can be identified as the point where the two linear equations intersect. In real-world problems, identifying the coordinates of this point may require estimation. For example:



A school club is planning to have some T-shirts printed.

- An online company will print the T-shirts for \$6 each, with \$15 for shipping.
- A local company will print the shirts for \$4.50 each after a \$50 set up fee.

The graph shows how the total cost for T-shirts at each company changes, depending on the number of T-shirts purchased.

From the graph, students are expected to estimate the solution to the system. In this case, this point falls at approximately (23, 153). This means that if the school club purchases about 23 T-shirts, the cost will be the same (around \$153) at both companies.

Learning from Mistakes Students may make the following mistakes:

- Incorrectly estimating systems of linear equations due to varying scales used on graphs

Stimulus

| Word Problem | Verbal Description | Chart/Table | Graph* |
|---------------------|--------------------|---------------|-------------------|
| Equation/Expression | Manipulatives | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

Item Types

| | | | |
|----------------------|---------------------------|-------------------------|--------------------------|
| Multiselect (2 pts) | Match Table Grid (2 pts) | Drag and Drop (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

Academic Vocabulary

coincide
 intersect/intersection
 linear equation
 parallel
 solution
 system of equations

Interesting Items

A.3(G) 2023 #30

TEKS Scaffold

| TEKS | Student Expectation |
|---------|---|
| 2A.3(F) | solve systems of two or more linear inequalities in two variables (S) |

A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

A.3(D)

(D) graph the solution set of linear inequalities in two variables on the coordinate plane

| | |
|---------|---|
| 7.10(B) | represent solutions for one-variable, two-step equations and inequalities on number lines (S) |
| 7.11(A) | model and solve one-variable, two-step equations and inequalities (R) |
| 6.9(B) | represent solutions for one-variable, one-step equations and inequalities on number lines (S) |

Stimulus

| Word Problem | Verbal Description | Chart/Table | Graph* |
|----------------------|--------------------|---------------|-------------------|
| Equation/Expression* | Manipulatives | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

Item Types

| | | | |
|----------------------|---------------------------|-------------------------|--------------------------|
| Multiselect (2 pts) | Match Table Grid (2 pts) | Drag and Drop (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing* (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

Content Builder (see Appendix for Tree Diagram)

- Graph the solution set of linear inequalities in two variables on the coordinate plane

Instructional Implications

Students are expected to graph linear inequalities on the coordinate plane. Instruction should include an introduction to two-variable inequalities where students are first asked to identify whether an ordered pair of x - and y -values is a solution.

For example, consider the inequality $x + y > 2$. To check if the ordered pair $(4, -1)$ is a solution, substitute the values for x and y into the inequality: $4 + (-1) > 2$ or $3 > 2$. Since this is a true statement, $(4, -1)$ is a solution (i.e. is in the solution set). Conversely, an ordered pair such as $(-2, 1)$ is not a solution, because $-2 + 1$ is not greater than 2.

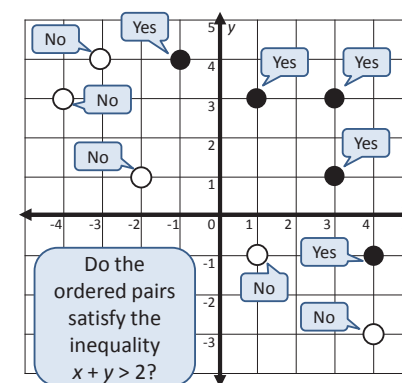
Student should then be asked to identify other solutions to the inequality by filling in or shading the ordered pairs that satisfy the inequality. In the sample graph shown, students may recognize a boundary line forming between the ordered pairs that satisfy the inequality (marked “yes”) and those that do not (marked “no”).

From this analysis, instruction should include directions for students on how to quickly identify the boundary line and shade the region that forms the solution set for a linear inequality.

In general, the chart below can be used for graphing linear inequalities in slope-intercept form.

| Inequality | Linear boundary | Slope | y-intercept | Shaded solution set |
|-----------------|-----------------|-------|-------------|---------------------|
| $y > mx + b$ | Dotted line | m | $(0, b)$ | Shade above |
| $y < mx + b$ | Dotted line | m | $(0, b)$ | Shade below |
| $y \geq mx + b$ | Solid line | m | $(0, b)$ | Shade above |
| $y \leq mx + b$ | Solid line | m | $(0, b)$ | Shade below |

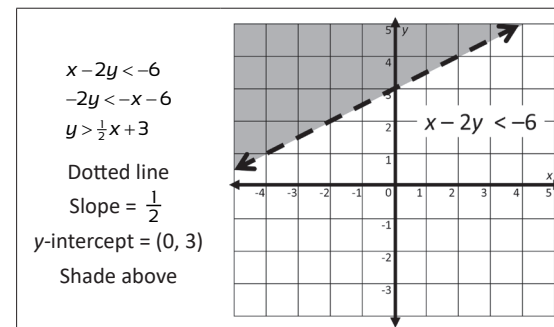
(continued)



Instructional Implications (continued)

Linear inequalities are not always written in slope-intercept form. For other types of inequalities, students are expected to solve for y in order to graph the solution set.

For example, graphing the inequality $x - 2y < -6$ requires subtracting x from both sides and dividing both sides of the inequality by -2 . Note that the inequality symbol must be reversed when dividing by a negative number.

**Learning from Mistakes**

Students may make the following mistakes:

- Confusing a negative and positive slope
- Confusing the x - and y -intercept
- Confusing when to use dotted lines or solid lines
- Confusing when to shade above or below the boundary line*
- Forgetting to reverse the inequality symbol when dividing both sides of an inequality by a negative number
- Having difficulty graphing solutions to systems of equations that are not represented in $y = mx + b$ format*

Academic Vocabulary

linear inequality
 solution set*

Interesting Items

A.3(D) 2022 #37
 A.3(D) 2021 #5
 A.3(D) 2021 #46

A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(H) write linear inequalities in two variables given a table of values, a graph, and a verbal description

Stimulus

| Word Problem* | Verbal Description | Chart/Table | Graph |
|----------------------|--------------------|---------------|-------------------|
| Equation/Expression* | Manipulatives | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

Item Types

| | | | |
|----------------------|---------------------------|--------------------------|--------------------------|
| Multiselect (2 pts) | Match Table Grid (2 pts) | Drag and Drop* (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

Academic Vocabulary

| | |
|------------------------------|-------------------|
| > (greater than) | inequality* |
| ≥ (greater than or equal to) | linear inequality |
| < (less than) | slope |
| ≤ (less than or equal to) | y-intercept |

Interesting Items

A.2(H) 2021 #30
A.2(H) 2016 #3

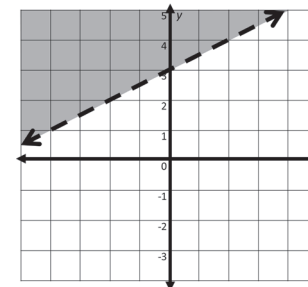
Role in Concept Development

| | |
|----------------------|---|
| Supports | A.3(D) graph the solution set of linear inequalities in two variables on the coordinate plane |
| Connection/Relevance | Students should move fluently between writing linear inequalities when given a table, graph, or verbal description and graphing solution sets of linear inequalities on a coordinate plane. |
| When to Teach | With A.3(D) |

Instructional Implications
Instruction should include writing linear inequalities from various representations. In conjunction with A.3(D), students should be able to create the graph when given a linear inequality. This standard requires students to complete the process in reverse (i.e. write the inequality from the graph). Instruction should include directions for students on how to quickly identify information from the boundary line (slope, intercept, dotted or solid) and shaded region (above or below the line) to write the linear inequality.

In general, the chart below can be used for writing linear inequalities in slope-intercept form.

| Linear boundary | Slope | y-intercept | Shaded solution set | Inequality |
|-----------------|-------|-------------|---------------------|--------------|
| Dotted line | m | $(0, b)$ | Shade above | $y > mx + b$ |
| Dotted Line | m | $(0, b)$ | Shade above | $y > mx + b$ |
| Solid Line | m | $(0, b)$ | Shade above | $y > mx + b$ |
| Solid Line | m | $(0, b)$ | Shade above | $y > mx + b$ |



For example, consider the graph at left. A student is expected to identify a dotted line with a slope of $\frac{1}{2}$ and a y-intercept of $(0, 3)$. Since the solution set is shaded above this line, the linear inequality can be written as $y > \frac{1}{2}x + 3$.

(continued)

Role in Concept Development (continued)

Instructional Implications

To write inequalities from verbal descriptions (word problems), students must first identify what the variables represent, then use the constants and constraints in the given situation to describe their relationship.

For example, consider the situation below.

At the food stand, tacos cost \$2 and burritos cost \$3. If Sam has \$15 how many tacos and burritos could he buy?

The two unknowns are the number of tacos and burritos (e.g., let x = the number of tacos and y = the number of burritos). The total cost would be determined by multiplying the number of tacos by \$2 and the number of burritos by \$3 and adding these products together. Since the total cost must be \$15 or less, the inequality can be written as $2x + 3y \leq 15$.

Learning from Mistakes

Students may make the following mistakes:

- Confusing the numbers or signs to use for constants like the slope and y-intercept
- Using the incorrect inequality symbol

A.3(H) **A.3 Linear functions, equations, and inequalities.** The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(H) graph the solution set of systems of two linear inequalities in two variables on the coordinate plane

Role in Concept Development

Supports

- A.3(D) graph the solution set of linear inequalities in two variables on the coordinate plane
- A.5(C) solve systems of two linear equations with two variables for mathematical and real-world problems

Connection/
Relevance

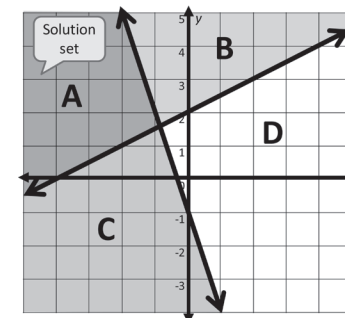
Students are expected to graph single linear inequalities. With this supporting standard, students graph two inequalities on the same grid to identify the solution set of the system.

When to Teach After A.3(D) and A.4(C)

Instructional
Implications

Instruction should include graphing a system of two-variable linear inequalities on the coordinate grid. Students should recognize the solution set as the region of the coordinate plane included in (or shaded by) both inequalities in the system.

For example, consider the system of two linear inequalities provided at right.

$$\begin{cases} y \geq \frac{1}{2}x + 2 \\ y \leq -3x - 1 \end{cases}$$


When graphed, the linear boundaries of the inequalities separate the coordinate plane into four sections, marked A, B, C, and D. The solution to the first inequality ($y \geq \frac{1}{2}x + 2$) consists of the sections above this line (marked A and B), and the solution to the second inequality ($y \leq -3x - 1$) consists of the sections below and to the left of the second line (marked A and C). The solution set to the system of both inequalities is the double-shaded section on the top left of this graph (marked A). Only in this region will the ordered pairs satisfy both inequalities simultaneously.

Learning from
Mistakes

Students may make the following mistakes:

- When graphing, incorrectly applying the slope or y -intercept when forming the linear boundary*
- Confusing whether the line is to be dotted or solid, and whether to shade above or below the line

Stimulus

| Word Problem | Verbal Description | Chart/Table | Graph |
|---------------------|--------------------|---------------|-------------------|
| Equation/Expression | Manipulatives | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

Item Types

| | | | |
|----------------------|---------------------------|-------------------------|--------------------------|
| Multiselect (2 pts) | Match Table Grid (2 pts) | Drag and Drop (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

Academic Vocabulary

linear inequality
slope
solution set
system of inequalities
 y -intercept

Interesting Items

A.3(H) 2021 #13

A.5 Linear functions, equations, and inequalities. The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:

(B) solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides

Role in Concept Development

Supports A.3(D) graph the solution set of linear inequalities in two variables on the coordinate plane

Connection/Relevance This standard supports students in solving equations and graphing inequalities.

When to Teach

- After A.5(A)
- Before/Prerequisite to A.3(D)

Instructional Implications Instruction should include solving a variety of linear, one-variable inequalities. Instruction can begin with a review of easier solution strategies learned in middle school (one-step and two-step inequalities in one variable) and should include a discussion of how inequalities are solved much like equations, but differ in that the inequality symbol must sometimes be reversed.

Stimulus

| | | | |
|----------------------|--------------------|---------------|-------------------|
| Word Problem | Verbal Description | Chart/Table | Graph |
| Equation/Expression* | Manipulatives | Diagram/Image | Number Line |
| Base Ten Blocks | Measurement Tool | Formula | Geometric Figures |

Item Types

| | | | |
|----------------------|---------------------------|-------------------------|--------------------------|
| Multiselect (2 pts) | Match Table Grid (2 pts) | Drag and Drop (1-2 pts) | Fraction Model (1-2 pts) |
| Hot Spot (1-2 pts) | Inline Choice (1-2 pts) | Number Line (1-2 pts) | Graphing (1-2 pts) |
| Text Entry (1-2 pts) | Equation Editor (1-2 pts) | Multiple Choice* (1 pt) | |

Academic Vocabulary

| | |
|------------------------------|-------------|
| > (greater than) | coefficient |
| ≥ (greater than or equal to) | distribute |
| < (less than) | inequality* |
| ≤ (less than or equal to) | solution* |

Interesting Items

A.5(B) 2016 #33

In Algebra I specifically, students are expected to solve multi-step inequalities that require the use of the distributive property and combining like terms. For example, consider the inequality $3(2x - 5) - 4x > 7x + 10$:

- Students must simplify the expression on the left-hand side of the inequality by using the distributive property and combining like terms.

$$3(2x - 5) - 4x > 7x + 10$$

$$6x - 15 - 4x > 7x + 10$$

$$2x - 15 > 7x + 10$$
- Students move the variables to the same side of the inequality by adding the opposite of one of these variable terms to both sides of the inequality.

$$-5x - 15 > 10$$
- Students isolate the variable by adding or subtracting constants on both sides, then dividing both sides by the coefficient of the variable.

$$-5x > 25$$

$$x < -5$$

Note that the only step that requires reversing the inequality symbol is the very last step, where both sides were divided by a negative number.

Learning from Mistakes Students may make the following mistakes:

- Making sign errors or computation mistakes when distributing or combining like terms*
- When multiplying or dividing both sides of an inequality by the same negative number, forgetting that the inequality sign must be switched (or reversed)*